Sec.- B Choose any 4 questions, choosing 1 question from each unit. Each question carries 15 marks.

1 (A) Derive the formulas for the Compton displacement and the kinetic energy of the reflected electron.

1 (B) In the Compton experiment, the Compton displacement of an X-ray at a scattering angle of 60° is 0.121 $\mathring{A}$. Find the value of Planck's constant.

2 (A) What is the de-Broglie hypothesis? For the wavelength λ associated with an electron, show that λ=h/2mE, where all symbols have their usual meanings. 2 (B) Find the de-Broglie wavelength for a 1 MeV proton.

3 (A) Derive the continuity equation ∂t∂P+∇.S=0 using the Schrödinger equation. 3 (B) Find the probability of finding a particle between x=0.45a and x=0.55a in a rigid box of length a, when the particle is in its first excited state.

4 (A) Find the reflection and transmission coefficients for a particle whose energy E is less than the height V0 of the potential ladder. 4 (B) An electron beam of energy 64 eV is incident on a potential barrier of height 100 eV and width 0.1 mm. Find the probability of the electron beam's transmission.

5 (A) Find the energy eigenvalues and eigenfunctions for a particle placed in a potential well of finite depth. 5 (B) Find the energy in the second excited state of a one-dimensional simple harmonic oscillator of angular frequency 1014 rad × s−1.

6 (A) Describe the Stern-Gerlach experiment and show that the electron spin has magnitude 1/2. 6 (B) If σ is the Pauli matrix and A is an arbitrary vector, then find σ·A.

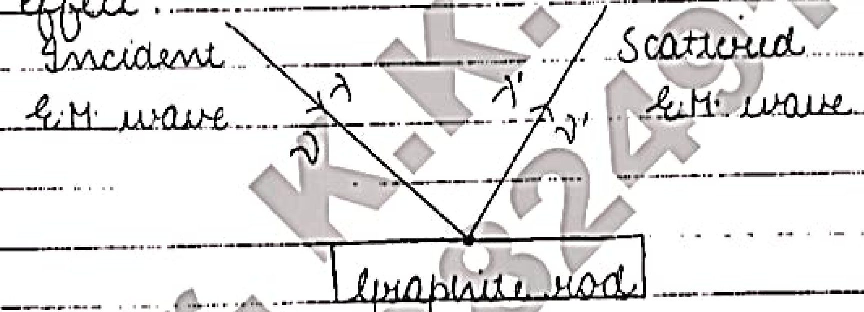
7 (A) What is the Zeeman effect? Derive the formula for Zeeman splitting. 7 (B) The moment of inertia of the NO molecule is 1.65×10−46 kg−m2. Find the energies of its first three rotational energy levels.

1 8 (A) Write a short note on - (i) Hund's rule (ii) Lande's gap rule (iii) Pauli's exclusion rule 8 (B) The force constant of the HCl molecule is 510 Nm−1. Calculate the values of its first three vibrational energy levels, E1, E2, and E3.

Compton Effect

* COMPTON EFFECT

According to compton scientist, "When electromagnetic waves are incident on graphide block then by interaction these waves are scattered. Frequency of scattered wave (ν') is less than the frequency of incident wave (ν) means wavelength of scattered wave (λ') is more than the wavelength of incident wave (λ). This displacement generate in wavelength is called compton displacement. This mathematical process defined in quantum mechanics satisfy the experiment. This effect is called compton effect.



$\nu > \nu'$

$\because \nu = \frac{c}{\lambda}$

$\lambda' > \lambda$

$$\boxed{\Delta\lambda = \lambda' - \lambda} \quad \text{---} (1)$$

Direction of Recoiled Electron

From momentum conservation law.

$mvc\sin\theta = h\nu'\sin\phi$ (In Y-direction) —— ①

$mvc\cos\theta = h\nu - h\nu'\cos\phi$ (In X-direction) —— ②

$\frac{①}{②}$

$$\frac{m\cancel{v}c\sin\theta}{m\cancel{v}c\cos\theta} = \frac{h\nu'\sin\phi}{h\nu - h\nu'\cos\phi}$$

$$\tan\theta = \frac{\cancel{h\nu'}\sin\phi}{\cancel{h\nu'}\left(\frac{\nu}{\nu'} - \cos\phi\right)}$$

MTR

$$\boxed{\tan\theta = \frac{\sin\phi}{\left(\frac{\nu}{\nu'} - \cos\phi\right)}} \quad —— ③$$

from eqⁿ Ⓐ

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\frac{\nu - \nu'}{\nu'} = \frac{h\nu}{m_0c^2}(1-\cos\phi)$$

$$\frac{\nu}{\nu'} - 1 = \frac{h\nu}{m_0c^2}(1-\cos\phi)$$

let $\frac{h\nu}{m_0c^2} = \alpha$.

$$\boxed{\frac{\nu}{\nu'} = 1 + \alpha(1-\cos\phi)} \quad —— ④$$

from eqⁿ ③ & ④.

$$\tan\theta = \frac{\sin\phi}{[1+\alpha(1-\cos\phi) - \cos\phi]}$$

$$\tan\theta = \frac{\sin\phi}{[(1-\cos\phi) + \alpha(1-\cos\phi)]}$$

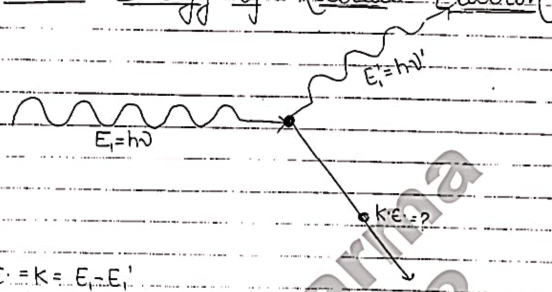
$$\tan\theta = \frac{\sin\phi}{(1-\cos\phi)(1+\alpha)}$$

$$\tan\theta = \frac{2\sin\phi/2\,\cos\phi/2}{(1+\alpha).\,2\sin^2\phi/2}$$

$$\tan\theta = \frac{\cot\phi/2}{(1+\alpha)}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{\cot\phi/2}{(1+\alpha)}\right]}$$

Kinetic Energy of Recoiled Electron



$$K.E. = K = E_1 - E_1'$$

$$K = h\nu - h\nu'$$

$$K = h\nu\left(1 - \frac{\nu'}{\nu}\right)$$

from eqn (4)

$$K = h\nu\left[1 - \frac{1}{1+\alpha(1-\cos\phi)}\right]$$

$$K = h\nu\left[\frac{1+\alpha(1-\cos\phi) - 1}{1+\alpha(1-\cos\phi)}\right]$$

MTR

$$\boxed{K = \frac{h\nu\alpha(1-\cos\phi)}{1+\alpha(1-\cos\phi)}} \quad \text{— (5)}$$

Case I If $\phi = 0 \Rightarrow \cos 0 = 1$

$$K = \frac{h\nu\alpha(1-1)}{1+\alpha(1-1)}$$

$$\boxed{K = 0}$$

If $\phi = \pi/2 \Rightarrow \cos\pi/2 = 0$

$$K = \frac{h\nu\alpha(1-0)}{1+\alpha(1-0)}$$

$$\boxed{K = \frac{h\nu\alpha}{1+\alpha}}$$

Case III If $\phi = \pi$, $\cos\pi = -1$

$$K = \frac{h\nu\alpha(1+1)}{1+\alpha(1+1)}$$

$$\boxed{K_{max} = \frac{2h\nu\alpha}{1+2\alpha}}$$

$$\because \alpha = \frac{h\nu}{m_0c^2}$$

$$K_{max} = \frac{2h\nu \cdot \frac{h\nu}{m_0c^2}}{1+2\left(\frac{h\nu}{m_0c^2}\right)}$$

$$K_{max} = \frac{2h^2\nu^2}{m_0c^2 + 2h\nu} \quad \text{— (6)}$$

$$K_{max} = \frac{h^2\nu^2}{\frac{m_0c^2}{2} + h\nu}$$

$$E_0 = m_0c^2$$

$$\boxed{K_{max} = \frac{E^2}{\frac{E_0}{2} + E}} \quad \text{— (7)}$$

Now, from eqn (6)

$$K_{max} = \frac{2h^2\nu^2}{m_0c^2 + 2h\nu}$$

$$\nu = \frac{c}{\lambda} \; ; \; \lambda_c = \frac{h}{m_0c} \Rightarrow \boxed{h = \lambda_c(m_0c)}$$

$$K_{max} = \frac{2\cdot\lambda_c^2(m_0c)^2 \cdot c^2/\lambda^2}{m_0c^2 + 2\lambda_c m_0c \cdot \frac{c}{\lambda}}$$

multiply $\frac{\lambda^2}{m_0c^2}$ b.s.

$$\boxed{K_{max} = \frac{2\lambda_c^2 m_0c^2}{\lambda^2 + 2\lambda_c\lambda}} \quad \text{— (8)}$$

In Compton experiment Compton shift is 0.121 Å at scattering angle 60^0. Calculate the Planck constant. **[Bikaner 2018]**

हल : कॉम्पटन प्रकीर्णन में कॉम्पटन विस्थापन

15

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

या

$$h = \frac{m_0 c \Delta\lambda}{(1 - \cos\phi)}$$

दिया है : $m_0 = 9.1 \times 10^{-31}$ kg , $c = 3 \times 10^8 ms^{-1}$

$$\phi = 60^0 \quad , \quad \cos 60^0 = \frac{1}{2} \quad , \quad \Delta\lambda = 0.121\ Å = 0.121 \times 10^{-10} m$$

$$h = \frac{(9.1 \times 10^{-31} kg)(3 \times 10^8 ms^{-1})(0.121 \times 10^{-10} m)}{(1 - 1/2)}$$

$$h = 6.61 \times 10^{-34}\ J.s$$

V.V. Imp

* Basic postulates of Quantum Mechanics

Quantum mechanics is defined by state function. This state function is called wave function. Wave function in 3-D is represented by $\psi(\vec{r},t)$. Wave function $\psi(\vec{r},t)$ is complex number. Therefore there is no physical meaning of $\psi(\vec{r},t)$ in the quantum mechanics. square of modulus of wave function $\psi(\vec{r},t)$ represent real number. so, in quantum mechanics this property is defined by probability density.

So, wave function:-

$$e^{-\frac{iE}{\hbar}t}$$

$$\boxed{\psi(\vec{r},t) = \psi(\vec{r}) \cdot e^{-\frac{iE}{\hbar}t}} \quad \text{——} (1)$$

probability density

$$\boxed{P = |\psi(\vec{r},t)|^2 = \psi^*(\vec{r},t)\,\psi(\vec{r},t)}$$

$$P = \psi^*(\vec{r})\, e^{\frac{iE}{\hbar}t} \cdot \psi(\vec{r})\, e^{-\frac{iE}{\hbar}t}$$

$$P = \psi^*(\vec{r})\,\psi(\vec{r})\, e^0$$

$$\because e^0 = 1$$

$$P = \psi^*(\vec{r})\,\psi(\vec{r})$$

$$\boxed{P = |\psi(\vec{r})|^2} \quad \text{——} (2)$$

It is clear from equation (1) & (2) that wave function $\psi(\vec{r},t)$ depends on position and time while probability density does not depend on time.

According to hypothesis of eqⁿ ① Max Born scientist wave function $\psi(\vec{r},t)$ present in eq ①, following properties should present

i) Wave function $\psi(\vec{r},t)$ should be finite

ii) Wave function $\psi(\vec{r},t)$ should be continous

iii) Wave function $\psi(\vec{r},t)$ should be single value

iv) Wave function $\psi(\vec{r},t)$ should be square integrable means, wave function $\psi(\vec{r},t)$ should be normalise

$$\boxed{\int |\psi(\vec{r},t)|^2 \, d\tau = 1}$$

v) First derivative of wave function should be continous

vi) In quantum mechanics average value is represented by expectation value.

$$\langle \hat{A} \rangle = \frac{\int \psi^*(\vec{r},t)\, \hat{A}\, \psi(\vec{r},t)\, d\tau}{\int \psi^*(\vec{r},t)\, \psi(\vec{r},t)\, d\tau}$$

$$\because \int \psi^*(\vec{r},t)\, \psi(\vec{r},t)\, d\tau = 1$$

$$\boxed{\langle \hat{A} \rangle = \int \psi^*(\vec{r},t)\, A\, \psi(\vec{r},t)\, d\tau}$$

vii) In quantum mechanics, any quantum state of system can be represented by in linear combination of their basis state.

$$\boxed{\Psi = \Psi_1 C_1 + \Psi_2 C_2}$$

viii) In quantum mechanics total energy is called hamiltonian energy. It is denoted by $\hat{H}$.

$$\hat{H} = \hat{T} + V(\hat{r})$$

$$\hat{H} = \frac{1}{2} m v^2 + V(\hat{r})$$

$$\boxed{\hat{H} = \frac{p^2}{2m} + V(\hat{r})}$$

It is called time independent operator.

* Time dependent energy operator is $\boxed{\hat{E} = i\hbar\partial/\partial t}$

ix) In quantum mechanics, quantum fundamental equation is defined by schrondinger scientist. This equation is called schrondinger differential equation. while in classical mechanics fundamental equation are defined by newton scientist. which is denoted by

$$\boxed{\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}}$$

so, schrondinger differential equation

$$\hat{H}\Psi = \hat{E}\Psi$$

$$\because \hat{H} = \frac{p^2}{2m} + V(\hat{r})$$

$$\hat{E} = i\hbar\partial/\partial t.$$

$$\left[\frac{\hat{P}^2}{2m} + V(\hat{r})\right]\psi = i\hbar\frac{\partial\psi}{\partial t}.$$

$$\frac{\hat{P}^2\psi}{2m} + V(\hat{r})\psi = i\hbar\frac{\partial\psi}{\partial t}.$$

$\because \hat{P} = -i\hbar\nabla$

$$\frac{(-i\hbar\nabla)^2\psi}{2m} + V(\hat{r})\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\hat{r})\psi = i\hbar\frac{\partial\psi}{\partial t}$$

NTR

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V(\hat{r})\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

NTR 3D

$$\boxed{-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V(\hat{r})\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

NOTE:-

In quantum mechanics defined probability $= \int P.dV$

$P =$ probability density $= \psi^*\psi = |\psi|^2$

$$\frac{\partial P}{\partial t} = \frac{i\hbar}{2m} \nabla . [\psi^* \nabla \psi - \psi \nabla \psi^*]$$

$$\frac{\partial P}{\partial t} + \frac{-i\hbar}{2m} \nabla . [\psi^* \nabla \psi - \psi \nabla \psi^*] = 0$$

$$\frac{\partial P}{\partial t} + \nabla \left[\frac{-i\hbar}{2m} (\nabla \psi^* \nabla \psi - \psi \nabla \psi^*) \right] = 0$$

$$\because \vec{S} = \vec{J} = \frac{-i\hbar}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*)$$ [Probability Current density]

$$\boxed{\frac{\partial P}{\partial t} + \nabla . \vec{S} = 0}$$

This is continuity equation.

A wave function of a particle in between $x = 0$ and $x = 1$ along x-axis is $\Psi = ax$. Calculate the probability of finding a particle in between $x = 0.45$ and $x = 0.55$.

हल: दिया है : $\Psi = ax \qquad 0 < x < 1$ 3B

3B $\Psi = 0 \qquad 1 < x < \infty$

$x = 0.45$ तथा $x = 0.55$ के बीच कण के पाये जाने की प्रायिकता

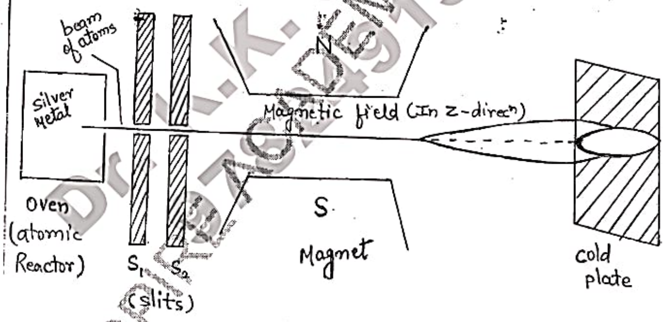
$$P = \int_{x_1}^{x_2} \Psi^* \Psi \, dx = a^2 \int_{0.45}^{0.55} x^2 \, dx$$

या
$$P = \frac{a^2}{3}[0.55^3 - 0.45^3] = 0.0251\, a^2$$

Stern - Gerlach Experiment =

In 1922, scientists Stern & Gerlach performed an experiment. Using this experiment, spin quantum no. (s) of e^- is calculated. And value of magnetic dipole moment is calculated.

② Experiment :-



In Stern-Gerlach experiment, a oven (atomic reactor) is used. silver metal is placed in the atomic reactor. High temperature and pressure are maintained of the silver metal in the atomic Reactor.

$$\vec{M}_l = -g_l \frac{e}{2m} \vec{L} \quad \text{(for orbital motion)} \quad - ⑤$$

$$\vec{M}_s = -g_s \frac{e}{2m} \vec{L} \quad \text{(for spin motion)} \quad ⑥$$

where-

$\vec{M}_s$ = spin magnetic moment

$\vec{S}$ = spin angular momentum

$\vec{M}_l$ = orbital magnetic moment

$\vec{L}$ = orbital angular momentum

g_l = orbital Lande's splitting factor = 1

g_s = spin Lande's splitting factor = 2

from eqn ⑥

$$\vec{M}_s = -g_s \frac{e}{2m} \vec{S}$$

$$\because g_s = 2$$

$$\boxed{\vec{M}_s = -\frac{e}{m} \vec{S}}$$

In z - direcn

$$(M_s)_z = -\frac{e}{m} S_z$$

$$\because S_z = \pm \frac{1}{2} \hbar$$

put value -

$$(M_s)_z = \pm \left(\frac{e\hbar}{2m}\right)$$

$$\boxed{(M_s)_z = \pm \mu_B} \quad - ⑦$$

where, μ_B = Bohr magneton

$$\mu_B = 9.27 \times 10^{-24} \ J T^{-1}$$

... स्तर की ऊर्जा ज्ञात कीजिये। 2025 2021

The moment of inertia of NO molecule is 1.65×10^{-46} kg-m^2. Determine the energy of its first three rotational energy levels.

हलः घूर्णी ऊर्जा स्तर के ऊर्जा मान 7B

$$E_r = \frac{\hbar^2}{2I} J(J+1)$$

$$\hbar = 1.054 \times 10^{-34}\, j - s \qquad , I = 1.65 \times 10^{-46} kg{-}m^2$$

$$E_r = \frac{(1.054 \times 10^{-34}\, j - s)^2}{2 \times (1.65 \times 10^{-46} kg - m^2)} J(J+1)$$

$$E_r = J(J+1) 3.37 \times 10^{23}\, J = J(J+1) \frac{3.37 \times 10^{23}\, J}{1.6 \times 10^{-19} C}$$

$$E_r = J(J+1) x 2.1 \times 10^{-4} eV$$

$J = 1$ के लिये $E_{r1} = 4.2 \times 10^{-4} eV$

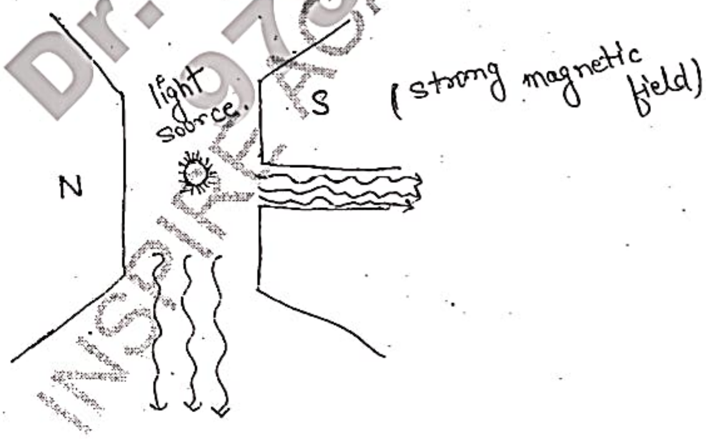
$J = 2$ के लिये $E_{r2} = 12.6 \times 10^{-4} eV$

$J = 3$ के लिये $E_{r3} = 25.2 \times 10^{-4} eV$

... ^{12}C एवं ^{16}O के द्रव्यमान क्रमशः 1.99×10^{-26} kg

Ziemann effect and Ziemann splitting =

In 1896 scientist Ziemann performed an experiment. Using this experiment, Ziemann explained splitting.



To explain ziemann effect, a light source is used. This light source is placed between strong magnetic field, due to which spectrum lines, are splitted into 3 spectrum lines. This effect is known as Ziemann effect.

And the splitting in spectrum lines is called Ziemann splitting.

Mathematical Analysis-

According to atomic physics, atom consists of orbitals and nucleus. e^- is present in the orbital and performs orbital motion in the orbital.

let e^- is moving in orbital of radius r. let linear velocity of e^- is $\vec{v}$ and time period is T.

From definition of electric current-

$$I = \frac{Q}{T}$$

for e^- $q = -e$

$$\boxed{I = \frac{-e}{T}} \quad -①$$

from eqⁿ ① it is clear that direcⁿ of electric current is opposite to motion of e^-.

Due to motion of e^- in orbital, magnetic field is produced along with electric field.

From definition of magnetic moment -

$$\mu = IA. \quad -②$$

where A = area , I = current

∵ Time = $\frac{\text{distance}}{\text{speed}}$

$$T = \frac{2\pi r}{v} \quad -③$$

put value of T in eqⁿ ①

$$I = -\frac{ev}{2\pi r} \quad -④$$

put value from eqⁿ ④ in eqⁿ ②

$$\mu = -\frac{ev}{2\pi r}A$$

∵ $A = \pi r^2$

$$\mu = -\frac{ev\,\pi r^2}{2\pi r}$$

$$\boxed{\mu = -\frac{evr}{2}} \quad -⑤$$

from definition of angular momentum -

$$\vec{L} = \vec{r} \times \vec{P}$$

∵ $\vec{P} = m\vec{v}$

$$\boxed{L = mvr} \quad -⑥$$

from eqⁿ ⑤

$$\mu = -\frac{evr}{2}$$

$$\mu = -\frac{e(mvr)}{2m}$$

put value from eqⁿ ⑥.

$$\mu = -\frac{eL}{}$$

If $\vec{\sigma}$ is a Pauli matrix and $\vec{A}$ is an arbitrary vector, then find $\vec{\sigma}.A$

हल: $\vec{\sigma}$ पाउली मैट्रिक्स है अत:

6B

$$\vec{\sigma} = (\hat{i}\sigma_x + \hat{j}\sigma_y + \hat{k}\sigma_z)$$

तथा

$$\vec{A} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)$$

$$\vec{\sigma}.\vec{A} = (\hat{i}\sigma_x + \hat{j}\sigma_y + \hat{k}\sigma_z).(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)$$

$$\vec{\sigma}.\vec{A} = \sigma_x A_x + \sigma_y A_y + \sigma_z A_z$$

$$\vec{\sigma}.\vec{A} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} A_x + \begin{vmatrix} 0 & -j \\ j & 0 \end{vmatrix} A_y + \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} A_z$$

$$\vec{\sigma}.\vec{A} = \begin{vmatrix} 0 & A_x \\ A_x & 0 \end{vmatrix} + \begin{vmatrix} 0 & -jA_y \\ jA_y & 0 \end{vmatrix} + \begin{vmatrix} A_z & 0 \\ 0 & -A_z \end{vmatrix}$$

$$\vec{\sigma}.\vec{A} = \begin{vmatrix} A_z & A_x - jA_y \\ A_x + jA_y & -A_z \end{vmatrix}$$