



R.K.

GROUP OF COLLEGES

Behind Kalwar Police Station, Kalwar, Jalandhar



ASSIGNMENT

R.K. VIGYAN P.G. MAHAVIDHYALAYA



Affiliated to University of Rajasthan, Approved by Govt. of Raj.)

Kalwar Road, Kalwar, Jaipur (Raj.)

Website : rkgroupofcollege.com , Mob. No. : 9314501146

E-mail : hrshreebalajieducationsamiti@gmail.com

B.A. / B.Sc. / B.Com.

ASSIGNMENT WORK / MIDTERM TEST

Session 20 - 20

Semester

Name of Student Manisha Choudhary .. Father's Name

Roll No. Enrollment No.

Year Semester

Question 1

If C_x and C_y are the lengths of the chords of curvature parallel to the axes at any point of the curve $y = ae^{x/a}$, then prove that:

$$\frac{1}{C_x} + \frac{1}{C_y} = \frac{1}{2a}$$

Question 2

If $v = f(x-y, y-z, z-x)$ then prove that:

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$$

Question 3

Evaluate the value of

$$\int_0^{\infty} \frac{x^8 (1-x^6)}{(1+x)^{24}} dx$$

Question 4 Find the volume generated by revolving an astroid about its major axis.

Solution 1

Here,

$$y = ae^{x/a}$$

$$y_1 = \frac{dy}{dx} = e^{x/a}$$

And,

$$y_2 = \frac{d^2y}{dx^2} = \frac{1}{a} e^{x/a}$$

Now, the chord of curvature parallel to the X-axis (C_x)

$$= 2P \sin \psi$$

$$= \frac{2y_1(1+y_1^2)}{y_2}$$

$$= \frac{2e^{x/a}(1+e^{2x/a})}{\frac{e^{x/a}}{a}}$$

$$= 2a(1+e^{2x/a})$$

The chord of curvature parallel to the Y-axis (C_y)

$$= 2P \cos \psi$$

$$= 2(1+y_1^2)$$

$$\Rightarrow \frac{2 \cdot (1 + e^{2x/a})}{e^{x/a}}$$

$$\Rightarrow \frac{2a (1 + e^{2x/a})}{e^{x/a}}$$

Therefore,

$$\frac{1}{C_x^2} + \frac{1}{C_y^2} = \frac{1}{4a^2 (1 + e^{2x/a})^2} + \frac{e^{2x/a}}{4a^2 (1 + e^{2x/a})^2}$$

$$= \frac{1}{2aC_x}$$

Therefore,

$$\boxed{\frac{1}{C_x^2} + \frac{1}{C_y^2} = \frac{1}{2aC_x}}$$

Solution

Let

Now

when

The

2
2

Solution 2

Let,

$$x - y = t_1, \quad y - z = t_2, \quad z - x = t_3$$

Now

$$V = f(t_1, t_2, t_3)$$

where ~~t_1, t_2, t_3~~ are functions of x, y , and z .

Therefore,

$$V = f(t_1, t_2, t_3)$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial t_1} \frac{\partial t_1}{\partial x} + \frac{\partial V}{\partial t_2} \frac{\partial t_2}{\partial x} + \frac{\partial V}{\partial t_3} \frac{\partial t_3}{\partial x}$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial t_1} (1) + \frac{\partial V}{\partial t_2} (0) + \frac{\partial V}{\partial t_3} (-1)$$

Since

$$t_1 = x - y$$

$$\therefore \frac{\partial t_1}{\partial x} = 1$$

$$\therefore t_2 = y - z$$

$$\therefore \frac{\partial t_2}{\partial x} = 0$$

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$$g) \quad t_3 = z - x$$

$$g) \quad \frac{\partial t_3}{\partial x} = -1$$

Therefore

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial t_1} - \frac{\partial v}{\partial t_3} \quad \text{--- (1)}$$

Similarly

$$v = f(t_1, t_2, t_3)$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial t_1} \frac{\partial t_1}{\partial y} + \frac{\partial v}{\partial t_2} \frac{\partial t_2}{\partial y} + \frac{\partial v}{\partial t_3} \frac{\partial t_3}{\partial y}$$

Therefore.

$$= t_1 = x - y$$

$$g) \quad \frac{\partial t_1}{\partial y} = -1$$

$$g) \quad \frac{\partial t_2}{\partial y} = 1$$

$$g) \quad \frac{\partial t_2}{\partial y} = 1$$

$$t_3 = z - x$$

$$c) \frac{\partial t_3}{\partial y} = 0$$

Therefore $\therefore$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial t_1} (-1) + \frac{\partial v}{\partial t_2} (1) + \frac{\partial v}{\partial t_3} (0)$$

$$\frac{\partial v}{\partial y} = -\frac{\partial v}{\partial t_1} + \frac{\partial v}{\partial t_2} \quad \text{--- (2)}$$

Similarly ∇

$$v = f(t_1, t_2, t_3)$$

$$\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t_1} \frac{\partial t_1}{\partial z} + \frac{\partial v}{\partial t_2} \frac{\partial t_2}{\partial z} + \frac{\partial v}{\partial t_3} \frac{\partial t_3}{\partial z}$$

Therefore

$$t_1 = x - y$$

$$c) \frac{\partial t_1}{\partial z} = 0$$

$$c) t_2 = y - z$$

$$c) \frac{\partial t_2}{\partial z} = -1$$

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$$t_3 = z - x$$

$$\Rightarrow \frac{\partial t_3}{\partial z} = 1$$

Hence ;

$$\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t_1}(0) + \frac{\partial v}{\partial t_2}(-1) + \frac{\partial v}{\partial t_3}(1)$$

$$\Rightarrow \frac{\partial v}{\partial z} = -\frac{\partial v}{\partial t_2} + \frac{\partial v}{\partial t_3} \quad \text{--- (3)}$$

Adding eqⁿ (1), (2) & (3)

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = \frac{\partial v}{\partial t_1} - \frac{\partial v}{\partial t_3} - \frac{\partial v}{\partial t_1} + \frac{\partial v}{\partial t_2} - \frac{\partial v}{\partial t_2} + \frac{\partial v}{\partial t_3}$$

$$\Rightarrow \boxed{\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0}$$

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Solution c3

$$d) \int_0^{\infty} \frac{x^8 (1+x^6) dx}{(1+x)^{24}}$$

$$e) \int_0^{\infty} \frac{x^8}{(1+x)^{24}} dx - \int_0^{\infty} \frac{x^{14}}{(1+x)^{24}} dx$$

$$f) \int_0^{\infty} \frac{x^{9-1}}{(1+x)^{9+15}} dx - \int_0^{\infty} \frac{x^{15-1}}{(1+x)^{9+15}} dx$$

$$\therefore \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx = B(n, n)$$

$$g) \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx = B(n, m)$$

$$\therefore B(m, n) = B(n, m)$$

Hence

$$h) \int_0^{\infty} \frac{x^{9-1}}{(1+x)^{9+15}} dx - \int_0^{\infty} \frac{x^{15-1}}{(1+x)^{9+15}} dx$$

$$= B(9, 15) - B(15, 9)$$

$$\therefore B(9, 15) = B(15, 9)$$

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Therefore

$$\Rightarrow B(9, 15) - B(15, 9) \\ \Rightarrow 0$$

Hence

$$\int_0^{\infty} \frac{x^8 (1-x^6)}{(1+x)^{24}} dx = 0$$

Solution = 4

The eqⁿ of the astroid is :

$$x^{2/3} + y^{2/3} = a^{2/3}$$

$$y = (a^{2/3} - x^{2/3})^{3/2} \quad \text{--- (1)}$$

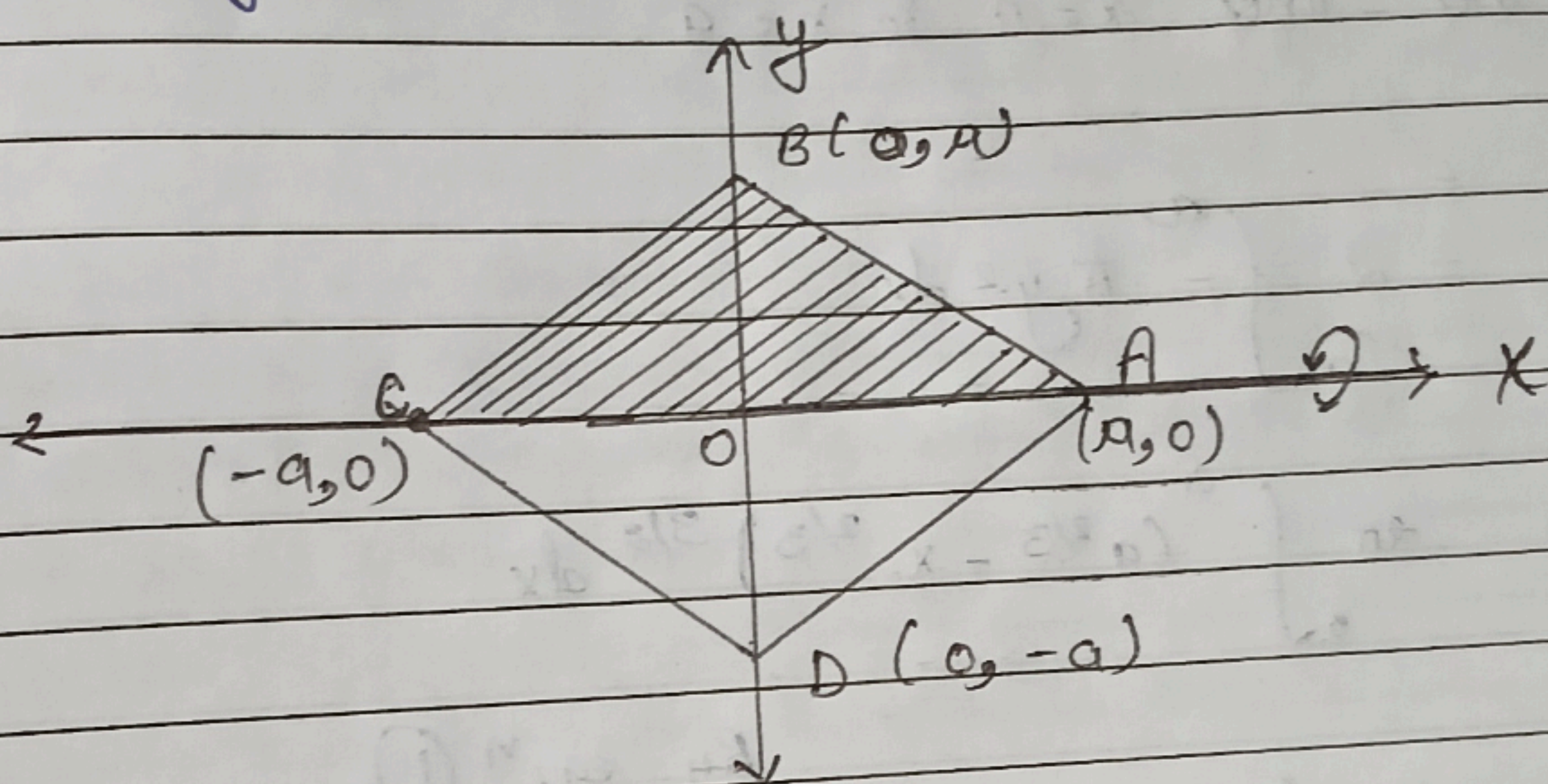


Figure (A)

Since the curve is symmetrical about both axes, the volume generated by revolving the region in the first quadrant (OAB) about the x-axis will be half of the total volume generated by revolving the entire region (ABCD) about x-axis.

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Again, Since the curve is symmetrical about the y-axis the volume the entire solid generated by revolving the area ABCA about the x-axis will be double the volume of the solid generated by revolving the area ABOA. The area ABOA is bounded by the arc AB, the x-axis, and the lines $x=0$ & $x=a$.

$$\therefore \text{Now} = 2 \int_0^a \pi y^2 dx$$

$$= 2\pi \int_0^a (a^{2/3} - x^{2/3})^{3/2} dx$$

By eqⁿ (1)

Now,

Let here,

$$x = a \sin^3 \theta$$

Given Then,

$$dx = 3a \sin^2 \theta \cos \theta d\theta$$

Let here

$$x = 0, \theta = 0$$

$$x = a, \theta = \frac{\pi}{2}$$

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Therefore, the required volume.

$$= 2\pi \int_0^{\pi/2} (a^{2/3} - a^{2/3} \sin^2 \theta)^3 \cdot 3a \sin^2 \theta \cos \theta d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} (1 - \sin^2 \theta)^3 \cdot \sin^2 \theta \cos \theta d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} (\cos^2 \theta)^3 \cdot \sin^2 \theta \cos \theta d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} \cos^6 \theta \cdot \sin^2 \theta \cos \theta d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} \cos^7 \theta \sin^2 \theta d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} \sin^2 \theta \cos^7 \theta d\theta$$

Here,

$$\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{\sqrt{\frac{m+1}{2}} \sqrt{\frac{n+1}{2}}}{2 \sqrt{m+n+2}}$$

By using this

$$= 6\pi a^3 \int_0^{\pi/2} \sin^2 \theta \cos^7 \theta d\theta$$

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$$= 6\pi a^3 \frac{\sqrt{3/2} \cdot \sqrt{4}}{2\sqrt{11/2}}$$

$$= 6\pi a^3 \frac{1/2 \cdot \sqrt{\pi} \cdot 3 \cdot 2 \cdot 1}{2 \cdot 9/2 \cdot 7/2 \cdot 5/2 \cdot 3/2 \cdot 1/2 \sqrt{\pi}}$$

$$= 6\pi a^3 \times \frac{16}{315}$$

$$= 6\pi a^3 \times \frac{16}{315}$$

$$= 2\pi a^3 \times \frac{16}{105}$$

$$= \frac{32\pi a^3}{105}$$

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Therefore, the volume of the solid in the
forms of Kupp Rupees, from unknown to known
according to Ampere's value, will be $\frac{32\pi r^3}{105}$.