# R.K.
## GROUP OF COLLEGE

Behind Kalwar Police Station, Kalwar, Jaipur (Raj.)

ASSIGNMENT

i) Solve this eq solvable for y

$p^2 - 9p + 18 = 0$

Soln → $p^2 - 9p + 18$ — ①

$= p^2 - 6p - 3p + 18$

$p(p-6) - 3(p-6)$

$(p-6)(p-3)$

$p = 3, 6$

$\boxed{\because \frac{dy}{dx} = p}$

$p = 3$

$= \frac{dy}{dx} = 3$

$= \int dy = \int 3dx$

$y = 3x + C$

$\boxed{y - 3x - C = 0}$ — ii

$$P = 6$$

$$\frac{dy}{dx} - 6$$

$$\frac{dy}{dx} = 6$$

$$\int \frac{dy}{\cancel{dx}} = \int 6\,dx$$

$$y = 6x + C$$

$$\boxed{y - 6x - c = 0} \quad - (ii)$$

$$(y - 3x - c)(y - 6x - C) = 0$$

$$\because p^2 - 9p + 18 = 0$$

$$\left(\frac{dy}{dx}\right)^2 - 9\left(\frac{dy}{dx}\right) + 18$$

$$\left(\frac{dy}{dx}\right)^2 - 9\frac{dy}{dx} + 18 = 0$$

Q: 2 solve the given equation

$$y + px = p^2 x^4$$

$\rightarrow$ $y + px = p^2 x^4$

$$y = p^2 x^4 - px \quad -(i)$$

$$\frac{dy}{dx} = x^4 \, 2p\frac{dp}{dx} + 4p^2x^3 - x\frac{dp}{dx} - p$$

$$p = x^4 \, 2p\frac{dp}{dx} + 4p^2x^3 - x\frac{dp}{dx} - p$$

$$p + p + x\frac{dp}{dx} = x^4 \, 2p\frac{dp}{dx} - x\frac{dp}{dx} + 4p^2x^3$$

$$2p + x\frac{dp}{dx} = x^4 \, 2p\frac{dp}{dx} + 4p^2x^3$$

$$2p + x\frac{dp}{dx} = 2px^3\left[x\frac{dp}{dx} + 2p\right]$$

$$= \left(2p + x\frac{dp}{dx}\right) - 2px^3\left[x\frac{dp}{dx} + 2p\right]$$

$$P = 6$$

$$\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = 6$$

$$\int \frac{dy}{d} = \int 6dx$$

$$y = 6x + C$$

$$\boxed{y - 6x - c = 0} \quad - iii)$$

$$(y - 3x - c)\ (y - 6x - c) = 0$$

$$[\because p^2 - 9p + 18 = 0]$$

$$\left(\frac{dy}{dx}\right)^2 - 9\left(\frac{dy}{dx}\right) + 18$$

$$\left(\frac{dy}{dx}\right)^2 - \frac{9dy}{dx} + 18 = 0 \quad \left[\because \frac{dy}{dx}\right.$$

$$\left(2p + x\frac{dp}{dx}\right) - 2x^3 p\left[x\frac{dp}{dx} + 2p\right] = 0$$

$$\left(2p + x\frac{dp}{dx}\right)\left(1 - 2x^3 p\right) = 0$$

$$\frac{dp}{dx}$$

$$\because \; 2p + x\frac{dp}{dx} = 0$$

$$x\frac{dp}{dx} = -2p$$

$$\frac{dp}{p} = \frac{dx}{x}$$

$$\int \frac{dp}{p} = -\int \frac{dx}{x}$$

$$\log p = -2\log x + \log c$$

$\log P = \log x^{-2} + \log e$

$\log P = \log c x^{-2}$

$$\boxed{P = c x^{-2}} \rightarrow 2$$

By eq$^n$ (1) and (2)

$$y = \left(\frac{c}{x^2}\right)^2 x^4 - \frac{c}{x^4} x$$

$$y = c^2 - \frac{c}{x}$$

$$\boxed{xy = c^2 x - c}$$

Q:3 Solve this linear diff. equation which complimatry function is exist in a intrigal.

$$x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$

Sol$^n$ = $x\dfrac{d^2y}{dx^2} - (2x-1)\dfrac{dy}{dx} + (x-1)y = 0$

$$\frac{d^2y}{dx^2} - \left(\frac{2x-1}{x}\right)\frac{dy}{dx} + \left(1-\frac{1}{x}\right) -(i)$$

$\because$ II order eq is $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$

$$P = -\left(2 - \frac{1}{x}\right)$$

$$Q = \left(1 - \frac{1}{x}\right)$$

Rule 1  $1 + P + Q = 0$

$$1 - 2 + \cancel{\frac{1}{x}} + 1 - \cancel{\frac{1}{x}} = 0$$

$$2 - 2 = 0$$

$$\boxed{0 = 0}$$

$$y = e^x$$

$$\boxed{y = Ve^x} - 2$$

Teacher's Signature......................................

$$\frac{dy}{dx} = Ve^{x} + e^{x}\frac{dU}{dx}$$

$$\frac{d^2y}{dx^2} = Ve^{x} + e^{x}\frac{dv}{dx} + e^{x}\frac{d^2V}{dx^2} + \frac{dU}{dx} e^{x}$$

$$\frac{d^2y}{dx^2} = Ve^{x} + 2e^{x}\frac{dU}{dx} + e^{2}\frac{d^2V}{dx^2} -$$

$\frac{d^2y}{dx^2}$ , Volue eq i)

$$Ve^{x} + 2e^{x}\frac{dU}{dx} + e^{x}\frac{d^2U}{dx^2} - \left(2 - \frac{1}{x}\right)\frac{dy}{dx} + \left(x - \frac{1}{x}\right)$$

$$Ve^{x} + 2e^{x}\frac{dU}{dx} + e^{x}\frac{d^2x}{dx^2} - \left(2 - \frac{1}{x}\right) \cdot (Ue^{x} + e^{x}\frac{dU}{dx}$$

$$+ \left(x - \frac{1}{x}\right)$$

$$\frac{d^2U}{dx^2} + \frac{1}{x}\frac{dU}{dx} = 0$$

$$\frac{d^2v}{dx^2} = \frac{-1}{x}\frac{dv}{dx}$$

$$\frac{d^2v}{dx^2} = \frac{-1}{x}\frac{dv}{dx}$$

$$\text{let}\cdot \frac{dv}{dx} = p$$

$$\frac{d^2v}{dx^2} = \frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{-p}{x}$$

$$\frac{dp}{p} = \frac{dx}{x}$$

$$\int \frac{dp}{p} = \int \frac{dx}{x} \quad [\because \text{Both Side intregation})$$

$$\log p = \log x + \log C$$

$$\cancel{\log} p = \cancel{\log} x \cdot C$$

$P_x = e^{c_1}$

$\frac{dv}{dx} \, x = c'$

$v = \log c' + c_x$

$v = \log e^x + c^2$

$v$

eq (2) $\boxed{y = ve^x}$

$y = \log e^x + c_2 \cdot e^x$

$y = \log e^x + e^x c^2$

$y = e^x(\log + c_2)$

Q: change of the linear variable (independent variable $x$ change to $z$

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 4x^3 \cdot \sin x^2$$

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - \frac{4x^3y}{x} = \frac{4x^3 \cdot \sin x^2}{x}$$

$$\frac{d^2y}{dx} - \frac{1}{x}\frac{dy}{dx} - 4x^2y = 4x^2\sin x^2 \quad (i)$$

$$P_1 = \frac{1}{x} \qquad Q_1 = -4x^2$$

$$R = 4x^2(\sin^2)$$

$$\text{let } \left(\frac{dz}{dx}\right)^2 = |Q| = |4x^2| = 4x^2$$

$$\left.\frac{dz}{dx}\right) = 2x \cdot \qquad \frac{d^2z}{dx^2} = 2$$

$$\int dz = \int 2x\,dx \qquad z = x^2 - (2)$$

With relation (2) the given differential eq (1)

$$\frac{d^2y}{dz^2} + P_1 \frac{dy}{dz} + Q_1 y = R_1 \quad -(3)$$

Where $P_1 = \dfrac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$

$$P' = \frac{2 + \left(\frac{-1}{x}\right) 2x}{\left(\frac{dz}{dx}\right)^2}$$

$$P_1 = \frac{2 - \frac{2x}{x}}{(2)^2} \qquad \frac{2-2}{4} = \frac{0}{4}$$

$$P_1 = 0$$

$$Q_1 = \frac{Q_1}{\left(\frac{dz}{dx}\right)^2} \qquad \frac{-4x^2}{4x^2} = -1$$

$$R_1 = \frac{R_1}{\left(\frac{dz}{dx}\right)^2} \qquad \frac{4x^3 \sin x^2}{4x^2} \qquad \frac{4x^3 \sin^2}{4x^2}$$

$P_1 = 2\sin x^2$

$\because z = x^2$

$R_1 = 2\sin z$

Put them in eq (3)

$$\frac{d^2y}{dz^2} - y = 2\sin z \quad -(4)$$

Solocution of eq (4) is —

$y = CF + I.F$

$$\frac{d^2y}{dz^2} - y = 2\sin z$$

$(D^2 - y) = \sin z$

$\because D = m$

$m^2 - 1 = 0$

$m = \pm 1$

$C.F = C_1 e^2 + C_2 e^{-2}$

या $C.F = C_1 e^2 + \frac{C_1}{2}$

$P.I = \frac{\theta(x)}{F(D)}$

$= \frac{2\sin 2}{D^2-1}$

$= 1 \; 2\sin^2 = D^2 = (-1)^2$

$\frac{1}{-2} \; 2\sin^2$

$P.I = -\sin 2$

Soloution of eq (4)

$y = C_1 e^2 + \frac{C_0}{2} - \sin 2$