R.K.

GROUP OF COLLEGE

Behind Kalwar Police Station, Kalwar, Jaipur (Raj.)

ASSIGNMENT

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B.A. / B.Sc. / B.Com.

ASSIGNMENT WORK / MIDTERM TEST

Session 20 .... - 20 ....

Semester ....

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Roll No. .... Enrollment No. ....

Year .... Semester ....

Question - 1

Find the number of students taking computer science and statistics both, but not mathematics, given the following data. In a group of total 90 Students, 40 student study mathematics, 50 students computer science, 60 students statistics, 10 students maths and computer science, 40 students maths and statistics and 10 study all three subjects?

Solution :-

Let A = Students taking maths
B = Students taking computer
C = Students taking statistics.

then

$|A \cup B \cup C| = 90$, $|A \cap B| = 10$
$|A| = 40$, $|A \cap C| = 40$
$|B| = 50$, $|A \cap B \cap C| = 10$
$|C| = 60$

So,
we have to find $|A' \cap B \cap C|$, so by principle of inclusion exclusion.

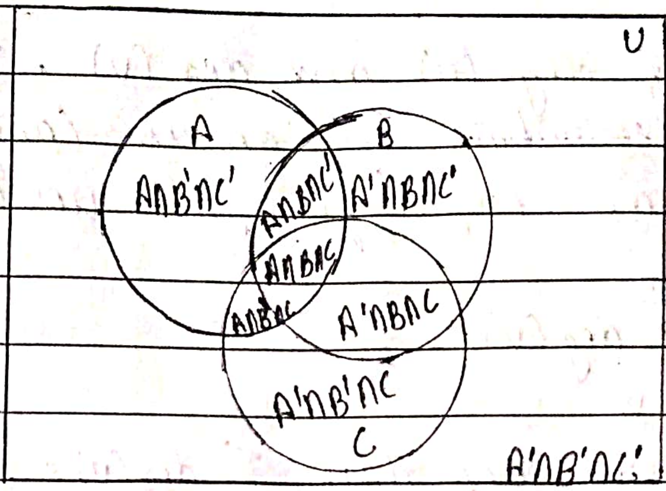
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Put all the values

$\Rightarrow 90 = 40 + 50 + 60 - 10 - |B \cap C| - 40 + 10$

$90 = 100 - |B \cap C|$

$|B \cap C| = 20$



A∪B∪C

It's clear that

$|B\cap C| = |A\cap B\cap C| + |A'\cap B\cap C|$

$20 = 10 + |A'\cap B\cap C|$

$|A'\cap B\cap C| = 10$

So number of students taking computer and statistics but not maths is 10.

Question 2

The sum of the degree of all the vertices in a graph is equal to twice the number of edges in the graph. that $\sum_{v\in V} deg(v) = 2e$

Proof :

Let $G = (V, E)$ be a graph with n vertices and number of edges are e vertices are $v_1, v_2, v_3 \ldots v_n$

Since each edge is incident with two vertices (v_1, u) say. So each edge contributes a count of 1 to each of deg (u) and deg (v). Therefore, e edges will contribute $2e$ degrees (one for each end vertices of an edges) for all vertices.)

that is

$$\sum_{v=v_1}^{v_n} \deg(v) = 2e$$

If graph have any loop then $\sum_{v \in V} \deg(v) = 2e$. Hence proved.

Question - 3

Let any disconnected graph G with n vertices and e edges has k components where every component is a tree then prove that

$$n = e + k.$$

Proof;

Let the graph G is having the following k - components $G_1, G_2, G_3 ---- G_k$ where every component is a tree.

Let G_i has n_i vertices $(1 \leq i \leq k)$, then clearly

$$n = n_1 + n_2 + ---- + n_k \quad —① $$

Now since all G_i is a tree, So mutually disconnected and having all the vertices are distinct. Since G_i is a tree, G_i has $n_i - 1$ edges.

So number of edges in a graph G

$$e = (n_1 - 1) + (n_2 - 1) + \text{-----} (n_K - 1)$$
$$e = (n_1 + n_2 + \text{---} n_K) - (1 + 1 + 1 + \text{---} k \text{ times})$$
$$e = n - k \quad \text{from equation -1}$$
$$\boxed{n = e + k}$$

Hence proof is proved.

Question-4

Find the optimum solution of the following LPP.

Solution max $Z = 5x_1 + 3x_2 + 0x_3 + 0x_4$

S.t. $3x_1 + 5x_2 + x_3 + 0x_4 = 15$

$5x_1 + 2x_2 + 0x_3 + x_4 = 10$

and. $x_1, x_2, x_3, x_4 \geq 0$

To change the problem in standard form, add two slack variables x_3 and x_4 we get.

The coefficient matrix A is:

$$A = \begin{bmatrix} 3 & 5 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{bmatrix} = (\alpha_1 \alpha_2 \alpha_3 \alpha_4) \text{ (Say)}$$

$$(\alpha_3 \; \alpha_4) = I_2$$

Taking the initial basis $B = [\alpha_3 \; \alpha_4]$ and construct the simplex table as follows

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Table - Ist

			C_j	5	3	0	0	min $\frac{X_{Bi}}{y_{i1}}$; $y_i > 0$
B	C_B	X_B	b	Y_1	Y_2	Y_3	Y_4	
α_3	0	X_3	15	3	5	1	0	$15/3 = 5$
α_4	0	X_4	10	$\boxed{5}$	2	0	1	$10/5 = 2$
	$(z_j - c_j) \rightarrow$			-5	-3	0	0	$10/5 = 2$
				↑			↓	

Entering vector (x_1)

Size $(z_1 - c_1) = -5$ is the most negative. So α_1 is the entering vector and

$$\min_i \left\{ \frac{X_{Bi}}{y_{i1}} ; y_i > 0 \right\} = \min \left\{ \frac{15}{3}, \frac{10}{5} \right\} = \frac{10}{2} = \frac{X_{B2}}{y_{21}}$$

So, $Y_{21} = 5$ is the key element and at $i = 2$, $\beta_2 = \alpha_4$ is departing vector and.

Now usual transformation, we obtain the next simplex table as follows :

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Table = 2

			C_j	5	3	0	0	$\min\left\{\frac{X_{Bi}}{y_{i2}}; y_{i2} > 0\right\}$
B	C_B	X_B	b	Y_1	Y_2	Y_3	Y_4	
α_3	0	x_3	9	0	$\boxed{19/5}$	1	$-3/5$	$9/19/5 = 2.37 \rightarrow$
α_1	5	x_1	2	1	$2/5$	0	$1/5$	$2/2/5 = 5$
$Z_j - C_j$				0	-1	0	1	

↑

Entering vector (α_2)

from above table it is clear that, α_2 is the key entering vector and α_3 is departing vector $y_{12} = \frac{19}{5}$ is key element.

Now usual transformation, the simplex table is obtained as :-

Table - 3

			C_j	5	3	0	0	$\min \frac{X_{Bi}}{Y_{ik}}; y_{ik} > 0$
B	C_B	X_B	b	Y_1	Y_2	Y_3	Y_4	
α_2	3	X_2	$\frac{45}{19}$	0	1	$\frac{5}{19}$	$\frac{-3}{19}$	
α_1	5	X_1	$\frac{20}{19}$	1	0	$\frac{-2}{19}$	$\frac{5}{19}$	
$Z_j - C_j$				0	0	$\frac{5}{19}$	$\frac{16}{19}$	

Since $(Z_j - C_j) \geq 0$. Hence the solution of problem is optimal.

$$x_1 = \frac{20}{19}, \quad x_2 = \frac{45}{19}, \quad x_3 = 0, \quad x_4 = 0$$

and max $Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{100 + 135}{19} = \frac{235}{19}$

Question 5

find the integers between 1 and 1000, which are not divided by 2, 3, 5 and 7.

Solution

first we find number of integers between 1 to 1000 which are not divisible by 2, 3, 5 and 7.

$$|A_1| = \left[\frac{1000}{2}\right] = [500] = 500$$

$$|A_2| = \left[\frac{1000}{3}\right] = [333.33] = 333$$

$$|A_3| = \left[\frac{1000}{5}\right] = [142.8] = 142$$

i.e. $x \leq x$

$$|A_1 \cap A_2| = \left[\frac{1000}{2 \times 3}\right] = \left[\frac{1000}{6}\right] = 166$$

$$|A_1 \cap A_3| = \left[\frac{1000}{2 \times 5}\right] = \left[\frac{1000}{10}\right] = 100$$

$$|A_1 \cap A_4] = \left[\frac{1000}{2 \times 7}\right] = 71$$

$$|A_2 \cap A_3| = \left[\frac{1000}{3 \times 5}\right] = 66$$

$$|A_2 \cap A_3| = \left[\frac{1000}{3 \times 5}\right] = 66$$

$$|A_2 \cap A_4| = \left[\frac{1000}{3\times 7}\right] = 47$$

$$|A_3 \cap A_4| = \left[\frac{1000}{5\times 7}\right] = 28$$

Now :-

$$|A_1 \cap A_2 \cap A_3| = \left[\frac{1000}{2\times 3\times 5}\right] = 33$$

$$|A_1 \cap A_2 \cap A_4| = \left[\frac{1000}{2\times 3\times 7}\right] = 23$$

$$|A_1 \cap A_3 \cap A_4| = \left[\frac{1000}{2\times 5\times 7}\right] = 14$$

$$|A_2 \cap A_3 \cap A_4| = \left[\frac{1000}{3\times 5\times 7}\right] = 9$$

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = |A_1| + |A_2| + |A_3| + |A_4| - |A_1 \cap A_2|$$
$$- |A_1 \cap A_3| - |A_1 \cap A_4| - |A_2 \cap A_3| - |A_2 \cap A_4|$$
$$- |A_3 \cap A_4| + |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4|$$
$$+ |A_1 \cap A_3 \cap A_4| + |A_2 \cap A_3 \cap A_4|$$
$$\Rightarrow |A_1 \cap A_2 \cap A_3 \cap A_4|$$

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = 500 + 333 + 200 + 142 - 166 - 100$$
$$- 71 - 66 - 47 - 28 + 33 + 23 + 14 + 9 - 4$$

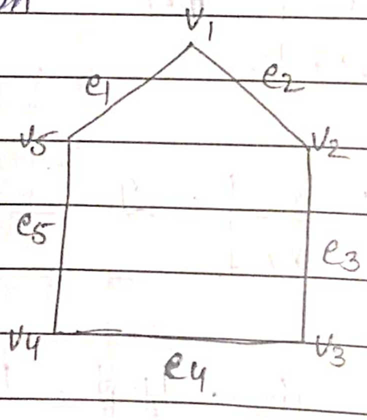
= 772

from 7 the divisible no of 2, 3, 5 and 7 have total 772 numbers.

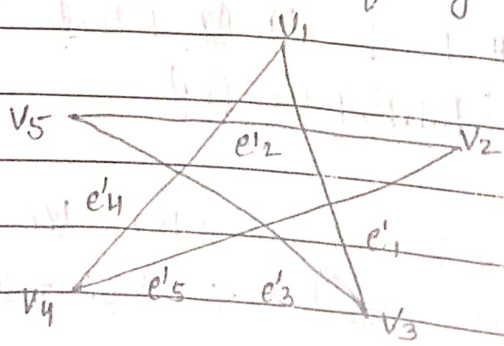
from 7 not divisible no. from 2, 3, 5 and 7 are
1000 − 772
$\boxed{228}$ Answer

Question - 6.
Show that cycle C_5 is self complementary graph.
Solution.

C_5 graph



Complementary graph $\overline{C_5}$ of graph C_5



C_5 and $\overline{C_5}$ are mutually isomorphic graph since

Number of vertices in $C_5 = 5$ and number of vertices in $\overline{C_5} = 5$ i.e. $|V_1| = |V_2|$

Number of edges in $C_5 = 5$ and number of edges in $\overline{C_5} = 5$ i.e

$$|E_1| = |E_2|$$

and degrees of corresponding vertices are equal so C_5 is self complementary graph.

Question - 7.

Find the dual of the following LPP.

Maximize. $Z_p = x_1 + 2x_2 - x_3$

s.t.

$$2x_1 - 3x_2 + 4x_3 \leq 5$$

$$2x_1 - 2x_2 \leq 6$$

$$3x_1 - 3x_3 \geq 4$$

$$x_1, x_2, x_3 \leq 0$$

Solution :

firstly, we change the problem into standard form. we have the objective function as maximization, the constraints must be in $\leq$ sign. In last inequality, change to the sign of '$\leq$' multiply both the sides by (-1) so, the standard form of the problem is

$$\max \; = \; Z_p = x_1 + 2x_2 - x_3$$

$$2x_1 - 3x_2 + 4x_3 \le 5$$

$$2x_1 - 2x_2 \le 6$$

$$-3x_1 + 3x_3 \le -4$$

$$x_1, x_2, x_3 \ge 0$$

Matrix form is

$$\max \quad Z_p = CX$$

$$\text{S.t} \quad AX \le b$$

$$X \ge 0$$

where $A = \begin{bmatrix} 2 & -3 & 4 \\ 2 & -2 & 0 \\ -3 & 0 & 3 \end{bmatrix}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 6 \\ -4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}$$

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The dual of this problem is

min $\quad Z_D = b^T W$

s.t $\quad A^T W \geq C^T$

and $\quad W \geq 0$

min $\quad Z_D = 5W_1 + 6W_2 - 4W_3$

s.t. $\quad 2W_1 + 2W_2 - 3W_3 \geq 1$

$\quad -3W_1 - 2W_2 \geq 2$

$\quad 4W_1 + 3W_3 \geq -1$

and $\quad W_1, W_2, W_3 \geq 0$

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