B.A/B.sc – III Geography Practical.
93

N.B. 1. There shall be 6 questions in written paper selecting at least two questions from each section. Candidates are required to attempt 3 questions selecting 1 question from each section. All question carry equal marks.

B.A/B.sc – III rd Geography Practical File

SYLLABUS

Section A

Definition, classification, uses and characteristic of map projection: (graphical constructions).

Conical projections:

1. with the one standard parallel
2. with two standard parallels
3. Bonne's
4. Polyconic

Cylindrical projections:

1. Equidistant
2. Equal Area
3. Mercator's, Universal Transverse Mercator (UTM)
4. Gall's Stereographic

Section B

Zenithal Projections: (Only Polar Case)

1. Equidistant
2. Equal Area
3. Gnomonic
4. Stereographic
5. Orthographic

Three dimensional diagrams: sphere, block pile, cube.

Section C

Plane table surveying: Equipments, procedure, traversing – open and closed traverse, methods- radial and intersection, concept of resectioning.

Height calculation using Indian pattern clinometer.

Recommended Readings:

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B.A/B.sc-III rd (Geography)
(Practical File.)

10 CHAPTER MAP PROJECTION

Meaning and Use

Map projection is a systematic drawing of parallel of latitude and meridians of longitude on a plane surface for the whole earth or a part of it on a certain scale so that any point on the earth surface may correspond to that on the drawing. The globe is true representative of the earth, which is divided into various sectors by the lines of latitude and longitude. The net-work of these is known as a *graticule.* A map projection, in other words, denotes the preparation of the graticule on a flat surface. Mathematically, the term projection means the determination of points on the plane as viewed from a fixed point. But in cartography it may not necessarily be restricted to perspective or "geometrical" projection. On the global the meridians and parallels are circles. When they are transferred on a plane surface, they become intersecting lines, curved or straight. If you stick a flat paper over the globe. It will not coincide with it over a large surface without being ceased. The paper will touch the globe only at one point, so that the other sectors will be projected over the plane in a distorted form. The projection with the help of light will give a shadowed picture of the globe which is distorted in those parts which are farther from the point where the paper touches it. The amount of distortion increases with the increase in distance from the tangential point. But only a few of the projections imply this perspective method. The majority of our projections represent an arrangement of lines of latitude and longitude in conformity with some principles so as to minimise the amount of distortion. With the help of mathematical calculation true relation between latitudes and longitudes is maintained. Thus various process of non-perspective projections have been devised.

The need for map projections arises from the very fact than an ordinary globe is rendered useless for reference to a small country. It is not possible to make a globe on a very large scale. Say, if you want to make a globe on a scale of one inch to a mile, the radius will be 330 ft. It is difficult to make and handle such a globe and uncomfortable to carry it in the field for reference. Not only topographical maps of different scales but also atlas and wall maps would not have been possibly made without the use of certain projections. So a globe is least helpful in the field for practical purposes. Moreover, it is neither easy to compare different regions over the globe in detail, nor convenient to measure distances over it. Therefore for different types of maps different projections have been evolved in accordance with the scale and purpose of the map.

Brief Historical Aspect

Our conception of the very ancient geography is obtained from writings of Homer and Hesiod.

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Homer's work was based on legend. Astronomical phenomena were vaguely used for such purposes as fixing of localities, marking of times of day and night and giving of sailing directions. Hesiod's work followed Homer's and made no further contribution to 'flat earth theory'. Crates of Malthus who died in about 145 B.C. constructed the first globe. About 600 B.C. Tales of Miletus came into prominence as the first person to predict successfully an eclipse of the sun. He is accredited with the introduction of what is now called Gnomonic projection. Anaximander wrote a formal treatise about nature and considered that the earth was a cylinder. Greeks had considerable familiarity with mathematics and geometry. About 540 B.C. Pythagorus of Samos maintained that the earth was a sphere. In 500 B.C. Hecataeus completed the correction of Anximander's map. Later on Herodotus and Democritus produced other world maps. Erathosthenes accepted the spherical shape of the earth and by measuring the altitude of the sun, deduced the circumference of the earth. His work was further developed by Hipparchus and Strabo. Strabo first stated the idea that a flat map cannot represent the features of the spherical earth and he suggested adjustment in meridians and parallels. Ptolemy made a map of the world on a net-work of curved meridians and parallels. From the time of Ptolemy until the 15th century, there was little real advance in geography. Ptolemy's projection approximates to simple conical projection and the Bonne's. Bonne's projection was further developed by Waldscecumuller in 1507. No advance took place during the Roman period. Germanus in 1466 produced the trapezifom map which later on lead to the Flamsteed projection. Fifty years later, Glatreanus made the first equidistant polar zenithal map. In 1554, Gerhard Kremer made a map of Europe on conical projection with two standard parallels. Kremer was also responsible for the Mercator's world map. In 1595 was published a book of maps in which the title 'Atlas' was used.

The subject of map projection has itself expanded in harmony with the new needs consequent upon the expansion of sea, land and air travel. Man's activity is largely confined to the outer surface of the earth and he must, therefore, design plans and maps regarding the configuration of the earth and its parts for his numerous requirements. Thus, a large number of projections were devised to represent the spherical surface on maps, charts and plans based on various map projections. There are now available a large number of projections out of which the geographer may choose the most suitable for his purpose. Broadly speaking, equatorial regions are satisfactorily mapped on a cylindrical projection, temperate regions on a conical projection, and polar regions on a zenithal projection; several modifications of these projections have been made according to the purpose of the map. An important development has been the use of modified polyconic or international (I/M) projection for mapping the world in 2,222 independent sheets which can be assembled together. From time to time many careful investigations of the properties of the projections have been made and many new ones proposed, some of which will be discussed in the following pages.

Classification of Map Projections

If you look at an atlas, or, any other set of maps, you will find a number of projections used therein. Map projection varies with the size and location of different areas on the earth's surface. While conical and zentithal projections are commonly used for mid-latitudes and polar regions, cylindrical projection are referred for equatorial lands. Not only that, projections also vary with the purpose of the map. While transferring the globe on a plane surface, the following facts should be kept in view : (*i*) Preservation of area, (*ii*) Preservation of shape, (*iii*) Preservation of bearing, *i.e.*, direction and distance. It is, however, very difficult to make such a projection even for a small country, in which all the above qualities may be well preserved. Any one quality may be thoroughly achieved by a certain map projection only at the cost of others. So the following groups of projections have been made according to the quality they preserve :

1. Equal area or homolographical projections.
2. Correct shape or orthomorphic projections.
3. True bearing or azimuthal projections.

In the first group of projections the graticule is prepared in such a way that every quadri-lateral on it may appear proportionately equal in area to the corresponding spherical quadri-lateral. It is, however,

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easier to make the area equal by ignoring the shape. For instance, a rectangle can be made equal in area to a parallelogram by keeping them on the same base and between the same parallels.

The second group of projection is known as *conformal projection*. It is relatively difficult to preserve the shape but for a very small area. Strictly speaking, only a few points of the sphere can be projected in their true form over a plane surface. In order to achieve the quality of *orthomorphism*, certain modification need to made. The scale is changed from point to point; it is true at one point in all the directions. It is possible to make some of the meridians and parallels true, *i.e.*, equal in length to the corresponding one on the globe. Meridians and parallels intersect each other at right angles on the globe. To make the projection conformal, certain devices are made so that they may cut one another of right angles over the graticule.

In the third group of projections, correct bearing or azimuths are preserved. This quality is well achieved in zenithal projections in which the sphere is viewed from a point laying either at the centre of the globe, or at the antipode of the central points or at infinity. The line of slight in every case is normal to the plane of projection at the central point. If the map is required to show all directions correctly, then the rectangular quality of the spherical quadrilaterial as well as the true proportion of its length and breadth must be maintained. In case, one wants to show all distances, correctly, no such map can be drawn on a plain sheet of paper.

There are various ways by which the globe can be projected over a surface. A flat paper may be tangent to the globe at one point and light may be kept at another point so as to reflect or project the lines of longitude and latitude on the plane. The shadowed graticule may be modified considerably by simply shifting the position of either the plane or light (See Fig. 255). In all cases some forms of geometrical or perspective projections shall be obtained. As the globe is viewed from a point vertically above it, these are called zenithal projections. They are also called animuthal becasue the bearings are all true from the central point where the plane is tangent to the sphere. The plane may touch the globe at the poles, or at the equator, or, at any point on the sphere between the poles and the equator. When it touches the globe at the poles, the projection is called *Polar Zenithal*; when it is tangent

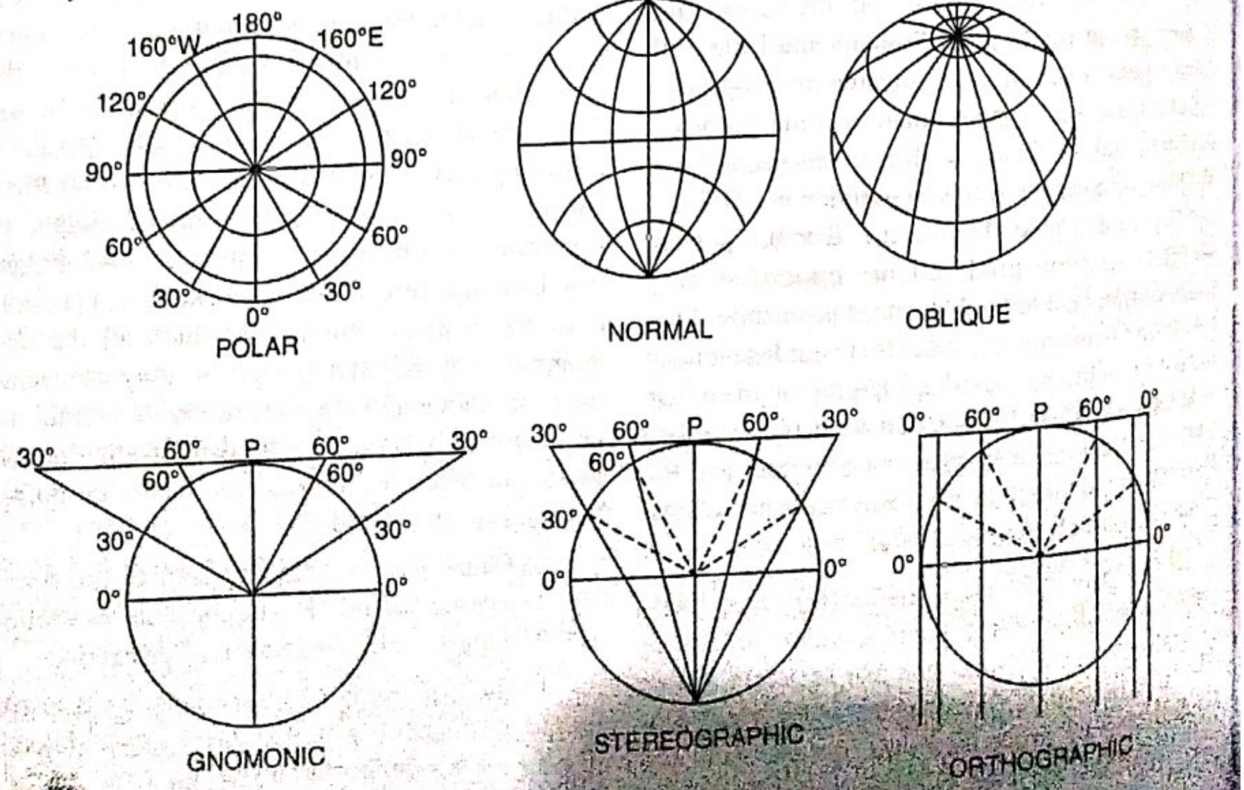

Fig. 255

at the equator, it is known as *Normal* or *Equatrial Zenithal;* and when it touches at any other point, it is called *Oblique Zenithal.* In all cases the view-point or the position of light must be on the diameter of the globe, or, on the diameter produced to infinity passing through the point at which the plane is tangent. This infinite line forms the locus of the position of light. In case the view-point is on the centre of globe, the projection is called *Gnomonic* or *Central Projection*; when the light is at the opposite end of the diameter, it is known as the *Sterographic Projections*; when the view-point is situated at infinity so that the *rays* may all be normal to the tangent plane, the projection is called *Orthographic.* Other positions of the light along the diameter may also be conceived and subsequently different names may be attributed to the projections thus derived; but their treatment will not be possible in the scope of this book.

There are other surfaces over which the sphere may be projected. After projection such surfaces may be cut open into flat surface. These *developable surfaces* include (*i*) cylinder and (*ii*) cone. When the graticule is prepared by imagining the surface of a globe projected on the surface of a hollow cylinder, it is called *Cylindrical Projection.* When the cylinder is unfolded into a flat surface, it gives a rectangular shape to the globe in which the meridians and parallels are represented as straight lines, intersecting each other at right angles, (Fig. 256). Here all the meridians are equal and parallel straight lines spaced at equal distances. Likewise, all the parallels of latitudes are also equal and parallel straight lines, but they are shaped at purposely calculated distances from the equator. The spacing of the parallels may be accurately calculated mathematically and it varies with different projections of this class, employed for varying purposes. In the equal area cylindrical projection, the distances between two parallels decrease proportionately towards the poles; while in Mercators, in which the shape and direction are preserved, the distances proportionately increase polewards. The cylinder may be made to circumscribe the sphere along the equator or along any other great circle, and you may have perspective forms too. But the most useful types will be the non-perspective equatorial projections in which the cylinder is conceived to circumscribe the globe along the equator.

A cone may be imagined to touch the globe of a convenient size along any circle (other than a great circle) but the most useful case will be the normal one in which the apex of the cone will lie vertically above the pole on the earth's axis produced and the surface of the cone will be tangent to the sphere along some parallel to latitude (Fig. 257). For the purpose the parallel selected is one along which the cone is tangential. If the selected parallel is nearer the pole, the vertex of the cone will be closer to it and subsequently the angle at the apex will be increasing proportionately. When the pole itself becomes the selected parallel, the angle at the apex will become 180°, and thus, the surface of the cone will be similar to the tangent plane of Zenithal Projection. On the other hand, when the selected parallel is nearer to the equator, the vertex of the cone will be moving farther away from the pole. In case the equator is the selected parallel, the vertex will be at an infinite distance and the cone will become a cylinder. Thus, the Cylindrical and Zenithal Projections may be regarded as special cases of Conical Projections (Fig. 258).

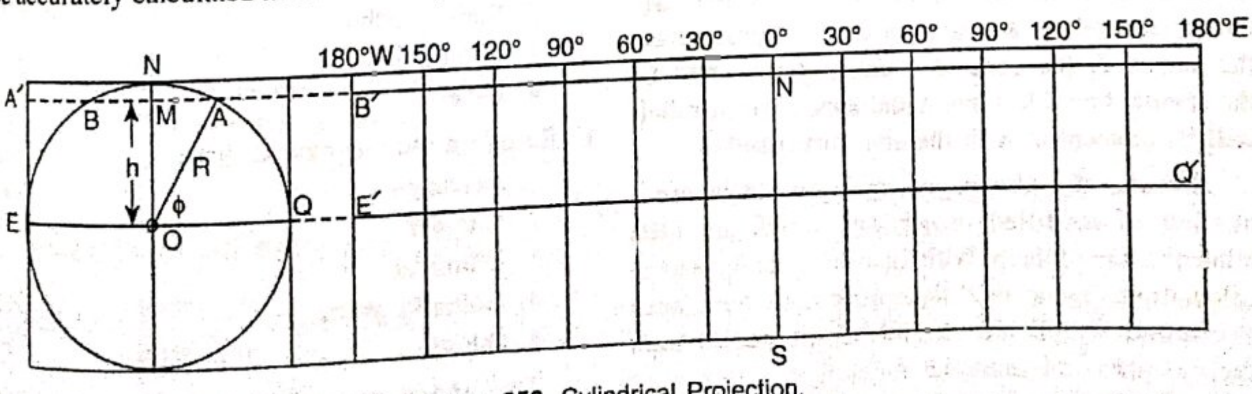

Fig. 256. Cylindrical Projection.

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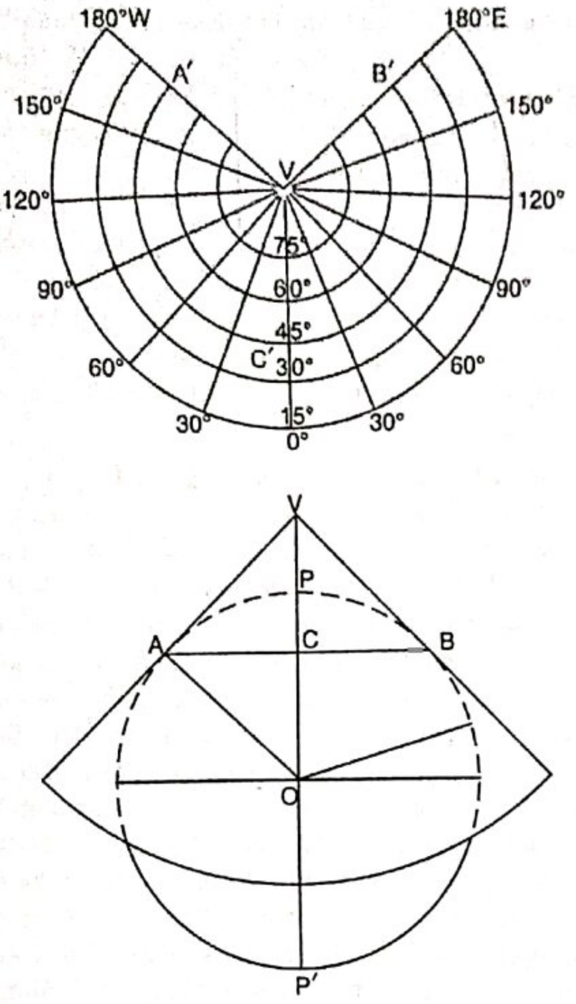

Fig. 257

The selected parallel is called the *standard parallel* because it truly corresponds to that of the globe and the scale along it is also true. This becomes an arc of circle after the cone is unfolded along a plane. There may be *one* or *two* standard parallels in conical projections. The axis along which the cone is flattened, form the central meridian of the map. Other medridians are straight lines radiating from the vertex of the cone at equal intervals, dividing the standard parallels into equal arcs. Other parallels will be concentric with the standard parallel.

Besides the above general types, there are a number of modified projections which are little related to any of them. With the help of mathematical calculations, some modified projections have been developed which are suitable for topographical survey maps, international maps and special atlas maps, etc. Even some of the conical projections have been modified to suit such maps; for instances, Bonne's and Polyconic Projections. All these modified projections may be called *Conventional Projections.*

Thus, now we have innumerable projections possessing one property or the other. The nature of these projections are so complex that they often posses one or more common properties. There is no projection which can be grouped in a single class. Moreover, if one attempts to obtain a rational classification of map projection, it will be rather difficult to acheive it. There can be as many classifications, as many bases. Hence following classifications may be suggested depending on different bases :

A. Based on the method of constructions :
   1. Persective.
   2. Non-perspective.

B. Based on the developable surface used :
   1. Conical.
   2. Cylindrical.
   3. Azimuthal or Zenithal.
   4. Conventional.

C. Based on the preserved qualities :
   1. Homolographic or equal area.
   2. Orthomorphic.
   3. Azimuthal or true bearing projections.

D. Based on the position of tangent surfaces :
   1. Polar.
   2. Equatorial or Normal.
   3. Oblique.

E. Based on the position of view-point or light :
   1. Gnomonic.
   2. Stereographic.
   3. Orthographic.
   4. Others.

F. Based on the Geometric shape :
   1. Rectangular.
   2. Circular.
   3. Elliptical.
   4. Butterfly shape.
   5. Others.

The above groups have their independent existence, but a single net can occur in more than

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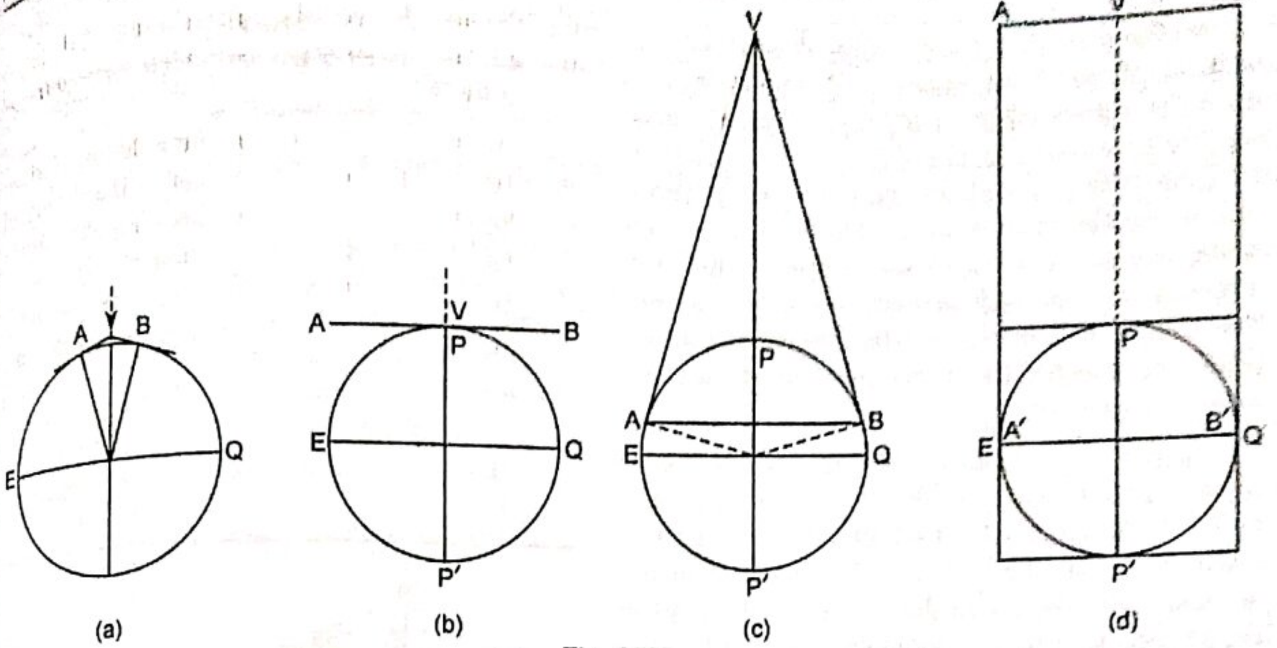

Fig. 258

one group, *i.e.*, an azimuthal projection is circular in nature in its polar case and may show correct area along with true azimuth of points from centre. This, no doubt, will be a non-perspective projection. Similar may be the case with other projections.

The Construction of Map Projections

In the construction of a graticule, both graphical and trigonometrical methods may be followed. The former is based on the elementary principles of geometry; it is simpler and approximately accurate, and it will suffice for the under-graduate students; while the latter makes use of trigonometrical formulae to calculate the radii and lengths of parallels and also their distances from the equator. In the case of difficult projections like the Mercator's and Mollweiode's, mathematical tables are also used. It is, however, essential to get an elementary knowledge of trigonometry so that the common projections may be easily followed. For the convenience of the students, various trigonometrical ratios are given as follows :

In the right-angled ΔABC (Fig. 259) :

$$AB/AC = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{P}{h} = \sin \angle ACB;$$

$$BC/AC = \frac{\text{Base}}{\text{Hyp.}} = \frac{b}{h} = \cos \angle ACB;$$

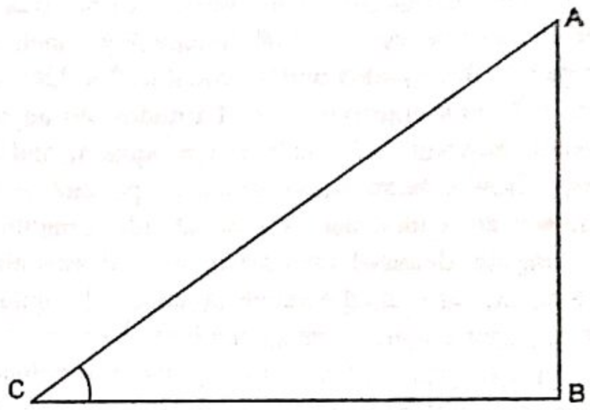

Fig. 259

$$AB/BC = \frac{\text{Perp.}}{\text{Base}} = \frac{p}{b} = \tan \angle ACB;$$

$$BC/AB = \frac{\text{Base}}{\text{Prep.}} = \frac{b}{p} = \cot \angle ACB;$$

$$AC/BC = \frac{\text{Hyp.}}{\text{Base}} = \frac{h}{b} = \sec \angle ACB;$$

$$AC/AB = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{h}{p} = \operatorname{cosec} \angle ACB.$$

In order to get the value of the ratios, Log tables may be consulted.

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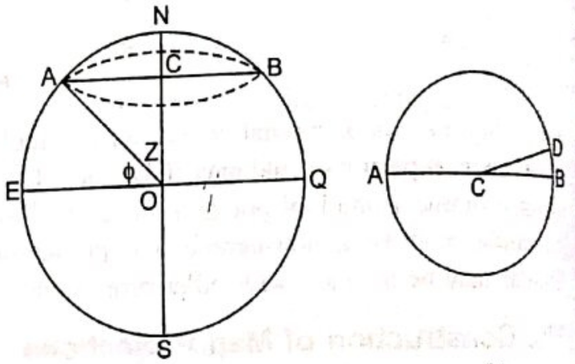
In the construction of Map Projections the scale is of much importance. In projection it is generally expressed by a fraction which is called the "Representative Fraction" (R.F.); such as 1 : 1,000,000 or 1 : 63,360, etc. The former means that one unit of the map is equal to the million units of the ground and if expressed in miles to the inch, it shows that one inch denotes one-inch-to-a-mile. The scale as R.F. is used in Map Projections so that it may be translated into any standard of measurement (Chapter 2).

According to Eratosthenes, the circumference of the earth is about 25,000 miles and the mean radius 3,960 miles (about 4,000 miles). As a round number, it may be regarded 250,000,000 inches because other figures in thousands will little affect ordinary scale. Thus a small globe of one inch radius will be 1/(250,000,000) the size of the earth. The length of the equator of the globe will be equal to the circumference, *i.e.*, $2\pi R$. In case $R = 1$ inch, the length of the equator will be equal to $2 \times 22/7 \times 1$, *i.e.*, 6.3 inch approximately. Latitudes are angular distances north and south of the equator, and the lines drawn, from those distances parallel to the equator are called parallels of latitude. Longitudes are angular distances measured east and west along the equator are called parallels of latitude. Longitudes are angular distances measured east and west along the equator the central meridian, and a meridian of longitude will be a line passing through the poles. All the meridians will be 360, ech drawn at a distance of one degree, and the parallels will number 181 when drawn at one degree interval. All the meridians will form 180 circles equal to the equator if two opposite meridians are combined together, so these are distinguished as *great circles*. All the parallels (between the equator and the poles) form small circles as their length decreases polewords. The length of one degree of latitude, for all practical purposes, may be taken as about 69 miles. It may, however, vary from 68.7 miles near the equator to 69.5 miles near the poles. The length of 1° of arc along various parallels, *i.e.*, the length of one degree of longitude, decreases towards the poles. This may be seen from the Table 1.

The length of each parallel may be easily calculated.

TABLE 1

Latitude	*1° of longitude*
0	69.6 miles
10	68.3 miles
20	64.8 miles
30	59.9 miles
40	52.9 miles
50	44.6 miles
60	34.8 miles
70	23.7 miles
80	11.8 miles
90	0.0 miles



(a) (b)

Fig. 260

In Fig. 260 (*a*), $OE = OA = R$ (Radius of the sphere) and the parallel AB is drawn at the distance of

$\angle EOA = \phi$

$\angle AON = Z$, the *co-latitude* $(90 - \phi)$

$\angle OAC = \phi$ as they are alternate angles.

In the right-angled ΔOAC,

$$\frac{AC}{AO} = \cos \phi$$

or $AC = AO \cos \phi$

$= R \cos \phi$ because $AO = R$

and $\frac{AC}{AO}$ sin Z, *i.e.*, $AC = AO \sin Z = R \sin$

$Z = R$. sin co.-lat.

Thus the length of the parallel AB will be $2\pi R \cos \phi$ or $2\pi R \sin Z$ because AC is the radius (r) of

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the parallel AB. Similarly the length of all the parallels can be calculated by the formula :

$$2\pi R \cos \phi$$

or

$$2\pi R \sin \text{co-lat.}$$

For cosine and sine, the mathematical table may be consulted.

The longitudinal distance between two meridians along the parallel AB (45°) can be obtained by dividing the length of the parallel AB by 360 and by multiplying the quotient by the distance (d) which may be 5°, or 10°, or 20°, etc. Thus when $R = 1''$, the length of the arc of the parallel AB for a distance of 10° will be

$$\frac{2\pi R \cos \phi \times d}{360} = \frac{2\pi R \cos 45° \times 10}{360}$$

$$= \frac{2 \times 22 \times 1 \times 0.71}{7 \times 36} = 0.12''.$$

The arc distance may, however, be found graphically also. Draw a circle with the radius AC. Let AB be the diameter. Draw CD making the $\angle DCB = 10°$. Now DB is the required longitudinal arc distance at the interval of 10° along the parallel AB [See Fig. 260 (*b*)].

Simple Conical Projection with one Standard Parallel

In the simple conical projection with one standard parallel, the cone is supposed to touch the sphere along the central parallel which is truly represented on the gratitude. The central parallel becomes the standard parallel because the scale is true only along this. Other parallels are drawn at their distances from it, but the scale along them is not correct. Thus, the scale is exaggerated north and south of the standard parallel. This makes it quite unsuitable for large areas, *i.e.*, areas with more than 20° of latitudinal extent. The meridians are drawn as straight lines converging on the vertex of the cone. They are equally spaced and intersect the parallels at right angles. They are also drawn at true distances measured along the standard parallel. The interval between two meridians may be obtained by dividing the standard parallel by 360, if the interval be 1°. The central meridian is first chosen, which runs through the middle of the area as a straight line. The distances between the parallels are marked along the central meridian, starting from the standard parallel. As all the parallels are drawn as arcs of concentric circle from a centre coinciding with the vertex of the cone, all the meridians including the central meridian become their radii, radiating from the vertex. So the scale along the meridians becomes correct.

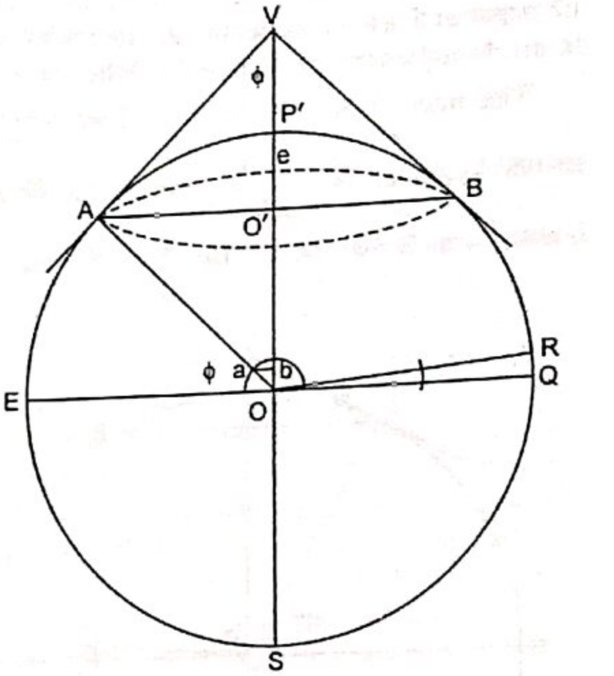

Fig. 261

In Fig. 261, let V be the vertex of the cone which is tangent to the sphere along the parallel AB. V is verically above P, the pole and lies on the prolonged polar axis of the sphere. If the cone is cut upon along VC, the standard parallel will become an arc of the circle drawn with radius VA, and centre V. Thus VA is the projected radius of the standard parallel ACB (Fig. 257). While its true radius (r) over the sphere is AO'. Now the length of VA and the standard parallel ACB may be calculated as follows :

In the right angled ΔVAO, $\angle AVO = \angle AOE = \phi$

$$\frac{VA}{OA} = \cot \phi$$

or $VA = OA \cot \phi = R \cot \phi$ as OA represent the radius (R) of the sphere.

Again in the right angled $\Delta AOO'$, $\angle OAO' = \phi$

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$\therefore \quad \frac{AO'}{AO} = \cos\phi$

or $\quad AO' = AO\cos\phi = R\cos\phi$

$\therefore \quad r = R\cos\phi$

$\therefore$ the length of the standard parallel $= 2\pi r = 2\pi r\cos\phi$.

Now a line *VO* may be drawn in the centre of the paper and with *V* as centre and radius *VA*, draw an arc to represent the standard parallel *ACB*.

The interval between the meridians along the standard parallel $\frac{2\pi R\cos\phi \times d}{360}$ when *d* is the given interval which may be 5°, 10°, 15°, 20°, etc.

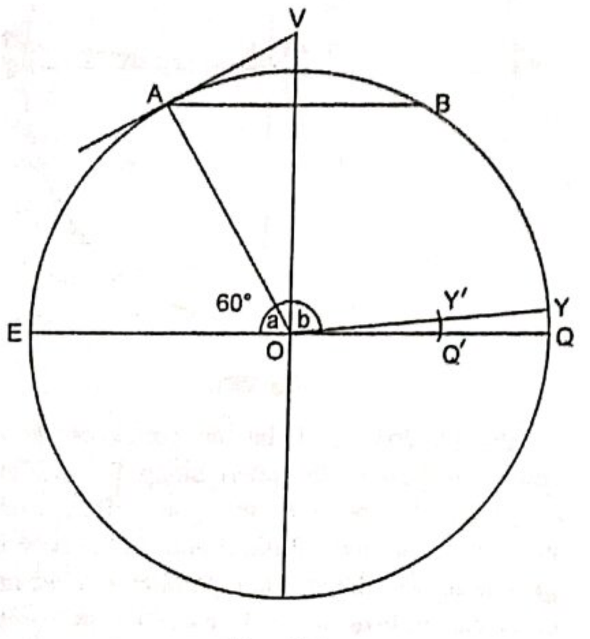

Fig. 262

From the point *C* mark-off the intervals along the standard parallel and draw straight lines from *V*, passing through the points of division. Then find out the distance apart between the parallels.

The distance $\frac{2\pi R \times d}{360}$ when *d* denotes the angular distance between the parallels. Mark-off the distance along the central meridian, starting from the point *C*; the distances between these points and *V* will from the radii of the respective parallels. Thus the projection may be completed for one complete hemisphere. But this projection is suitable only for narrow zones (up to 20° of latitudinal extent) lying in mid-latitudes. Moreover, it is more suitable for the cool temperate regions, e.g., Baltic States, Ireland, etc. which can be, more or less, accurately represented.

EXAMPLE

To construct a graticule on simple conic projection on 1 : 25,000,000 scale at the interval of 5° for an area stretching between 50°N–70°N and 5°E–35°E.

Let the standard parallel be 60°N which will be the central parallel of the area and 20°E be the central meridian. The radius of the sphere on the given scale

$$= \frac{250,000,000}{25,000,000} = 10''.$$

Graphical Construction

Draw a circle *AEQ* with 10″ radius. From its centre *O* draw *OA*, making the ∠*EOA* = 60°. From the point *A*, draw *AV* as tangent to the circle at *A* to meet the polar diameter produced at *V*. Now *VA* is the projected radius of the 60° north latitude line. Make the ∠*rOQ* = 5°, the given interval between the parallels. *Qr* is the true distance between two parallels. *Qr* is the true distance between two parallels at 5° interval. With centre *O* and radius *Qr* describe a semi-circle which intersect *OA* at the point *a*. From a draw ab parallel to *EO*, the line *ab* meeting *OV* at *b*. Thus *ab* is the longitudinal distance between two meridians at the interval of 5° along standard parallel (Fig. 262).

Then draw *VO* in the centre of the paper. With centre *V* and radius *VA* draw the arc *ACB*. From *C* mark-off the points *Y*, *Z*, *M*, *L* along *VO*, making, *CY*, *CM*, *YZ* and *ML* equal to *Qr*. With centre *V* draw concentric arcs passing through *L*, *M*, *Y*, *Z* respectively. Similarly mark along the arc *ACB* longitudinal points at distances equal to *ab*. Draw straight line from *V* passing through the points thus marked. In this way complete the graticule for the area.

Trignometrical Construction

The projected radius of the standard parallel $= R\cot 60° = 10 \times 0.58 = 5.8''$.

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The length of the standard parallel

$= 2\pi r \cos 60°$

$$= \frac{2 \times 22 \times 10 \times 0.5}{7} = 31.4''$$

The distances between the two meridians along the standard parallel

$$= \frac{31.4}{360} \times 5 = 0.44''$$

The true distance between the two parallels at 5° interval

$$= \frac{2\pi R}{360} \times 5 = \frac{55}{63} = 0.87''.$$

The construction may now be completed as in the foregoing, to produce the require graticule (Fig. 263).

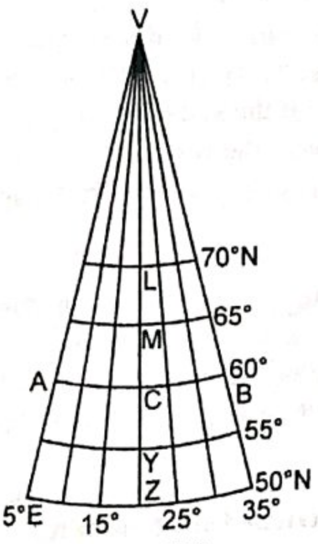

Fig. 263

Simple Conic Projection with two Standard Parallels

In the simple conic projection with two standard parallels, two of the circles of the cone are equal to two of the parallels of latitude of given area. Along these two lines of latitude, the scale is correct. This is why they are called standard parallels. Unlike the simple conic with the one standard parallel here the code neither touches the sphere along the parallels, not cuts through the sphere along them. Instead two circles of the cone correspond to the two respective parallel of the globe and form an ordinary cone independent of the globe [Fig. 264(*a*)]. These are so selected as to cover two-thirds of the latitudinal extent of the map. In this way errors are uniformly distributed on account of which a wider extent of area may be represented. This is, indeed, an improvement over the simple conic with one standard parallel; otherwise the projection has the same properties. It is misleading to call it the Secant conic projection as the distance between the standard parallels does not equal the true Secant distance between them. Any straight line cutting the circumference at two points represents a Secant of a circle. While in the projection the use is made of the arc distance so that the parallel may be set apart proportionately the same distances as on the globe [Fig. 264 (*b*)].

In Fig. 264(*a*) suppose *AC* and *BD* are actual radii of the standard parallels and *VD* be the produced

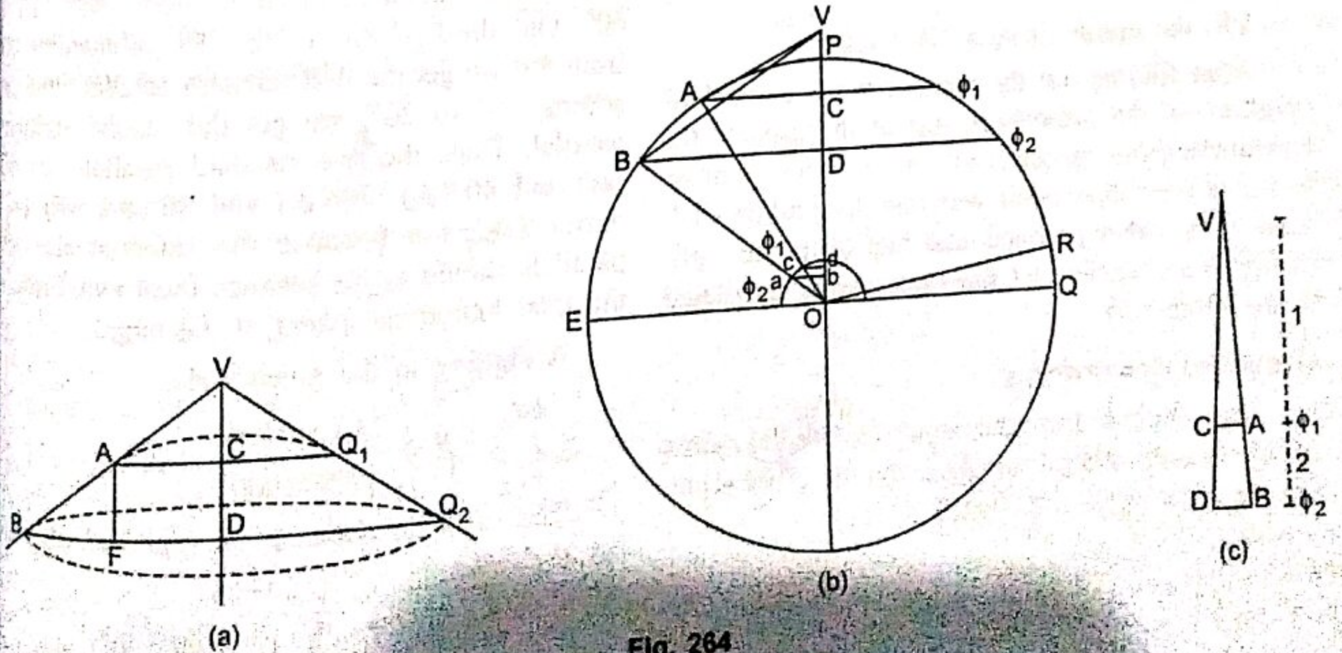

Fig. 264

axis of the globe. The distance between the standard parallels is represented by a straight line *AB* which on the sphere is an arc—the true distance between them. *BA* is produced to meet *V*, the vertex of the cone, $AB = \frac{2\pi R \times d}{360}$ when *d* is the interval between the standard parallels. Now the main problem is to find the radii of the standard parallels. In the right-angled ΔQBD and *OAC* in Fig. 264(*b*).

$$\frac{DB}{OB} = \cos\ \phi_2;$$

or $\quad DB = R\cos\phi_2$ when *OB* is *R* of the sphere;

$$\frac{CA}{OA} = \cos\ \phi_1;$$

or $\quad CA = R\cos\phi_1$ when OA is *R* of the globe.

Now, in Fig. 264(*a*) *VA* : *VB* :: *CA* : *DB* or, *VA* : *AB* :: *CA* : *BF* because $AB = VB - VA$ and $BF = DB - CA$ when *AF* is perpendicular to *BD*.

$$VA\ .\ BF = AB\ .\ CA$$

VA, the radius of the parallel

$$\phi = \frac{AB.CA}{BF}$$

$$= \frac{\frac{2\pi R}{360}\times d\times R\cos\phi_1}{R\cos\phi_1 - R\cos\phi_2}$$

VB, the radius of $\phi_2 = VA + AB$

After finding out the radii of the two standard parallels on the projection, rest of the process for constructing this projection is, more or less, similar to that of the simple conic with one standard parallel. That is the other parallels and meridians are equidistant and as such other spacings may be calculated in the same way.

Graphical Construction

Draw a circle from the centre *O* with the radius of the reduced sphere, calculated on the given scale. On the scale of 1 : 25,00,000.

$$R = \frac{250,000,000}{25,000,000} = 10''.$$

Draw *OA* and *OB* at angular distances denoting the standard parallels, ϕ_1 and ϕ_2 and *OR* at given interval, say 10°. Describe a semi-circle from *O* with the radius *QR* to cut *OB* and *OA* at *a* and *c*. From *a* and *c* draw *ab* and *cd* parallel to *EQ* to meet the polar diameter at *b* and *d*. Thus *cd* and *ab* represent the spacing between the meridians at the given interval along the parallels ϕ_1 and ϕ_2 respectively; while *OR* shows the interval between two parallels [Fig. 264(*b*)].

Now draw the central meridian *Vb*; on it mark *AB*, the arc distance as straight line which is equal to $(\phi_1 - \phi_2)/d$ times the length of *OR* when *d* is the given interval. From *A* and *B*, erect perpendicular *AC* and BD, making $BD = ab$ and $AC = cd$. Join *DC* and the make it meet *VB* at *V*. Then *VA* and *VR* represent the radii of the two standard parallels ϕ_1 and ϕ_2 respectively [Fig. 264(*a*)]. With these radii, draw the arc of the standard parallels from the centre *V* and complete the rest of the construction as in the simple conic with one standard parallel.

EXAMPLE

Construct a graticule on the simple conic projection with two standard parallels on a 1 : 25,000,000 scale for an area extending from 20°N to 80°N and from 0° to 80°W at the interval of 10°.

Trigonometrical Construction

The latitudinal extent of the area = 80° − 20° = 60°. One-third of this is 20°. By subtracting 20° from 80° we get the first standard parallel while by adding 20° to 20°, we get the second standard parallel. Thus, the two standard parallels are 60° (ϕ_1) and 40°(ϕ_2). 70°(ϕ_1) and 30°(ϕ_2) will be a better selection because the selected standard parallels should cover between them two-thirds of the total latitudinal extent of the map.

According to the given scale,

$$R = \frac{250,000,000}{25,000,000} = 10''.$$

The distance between the standard parallels, say *d*.

$$= \frac{2\times 22\times 10\times(60^\circ - 40^\circ)}{360\times 7} = 3.5''$$

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r_1, the radius of the 60° parallel

$$= \frac{d \times R \cos \varphi_1}{R \cos \varphi_2 - R \cos \varphi_1}$$

when d is the distance between

$$= \frac{3.5 \times 10 \times 0.5}{(10 \times 0.77) - (10 \times 0.5)} = 6.5''$$

$\therefore$ r_2, the redius of the 40° parallel
$= 6.5'' + 3.5'' = 10.0''$.

The describe the arc of ϕ_1 and ϕ_2, from the point V lying on the central meridian of 40°W passing through the points B and A. From A and B mark-off the points along the central meridian at the interval of $\frac{3.5 \times 10}{20}$ *i.e.*, 1.75″.

The length of the 60° parallel

$$= 2\pi R \cos 60$$

$$= \frac{2 \times 22 \times 10 \times 0.5}{7}$$

$\therefore$ the longitudinal interval

$$= \frac{2 \times 22 \times 10 \times 0.5 \times 10}{7 \times 360} = 0.87''$$

Mark-off the points along 60°N parallel at the interval of 0.87″. Draw the meridians joining these points with V. Thus the required graticule $MNLP$ is obtained (Fig. 265). This may also be drawn graphically as discussed in the foregoing.

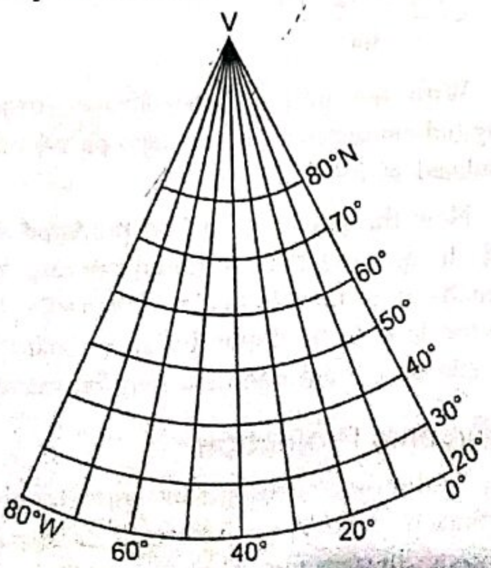

Fig. 265

The projection is, therefore, not very difficult to construct. It is neither equal area nor orthomorphic. But in atlases it has been usually adopted for the continents like Europe, Asia and North America, etc. It is, however, one of the most suitable projections for mid-latitude countries with small latitudinal extent.

Bonne's Conical Projection

This is modified conical projection designed by Rigobert Bonne, a French Cartographer. In this system all parallels are true to scale but, like the simple conic with one standard parallel, it has only one selected parallel (the standard parallel) drawn on definite radius which is the cotangent of the selected parallel multiplied by the radius of the reduce sphere ($R \cot \phi$). The selected parallel varies with different areas, as it governs the curvature of other parallels. If the main mass of an area is nearer the pole, the selected parallel will be one nearer the pole and if it be close to the equator, the selected parallel will be one nearer the equator. Supposing for a continent like Asia, if 80°N is the selected parallel, then the radii for other parallels will be so short as to compress the shape of the continent on the map. For it, 40°N will be better because in this case, to start with, the radius will be longer and the successive radii of other parallels will be conformable so that the map will appear a little distorted. All other parallels are concentric arcs whose radii are found by marking-off divisions along the central meridian true to scale. The central meridian is a straight line. All other meridians are regular curves drawn by joining the points marked along the parallels at true distances according to the given interval. Every quadrangle, thus formed on the graticule, is equal in area to the correspoinding quadrangle of the sphere because all the parallels are drawn true to scale and they are truly spaced from each other (Fig. 266). This is why it is an equal area projection. The projection is conformal only along the central meridian. The amount of distortion increases towards the margins of the map. It may be used for drawing one hemisphere but its lateral distortion restricts it suitability for continents separately. In atlases it has been commonly used for the maps of all continents except Africa. The map of Africa is generally drawn on Sinusoidal

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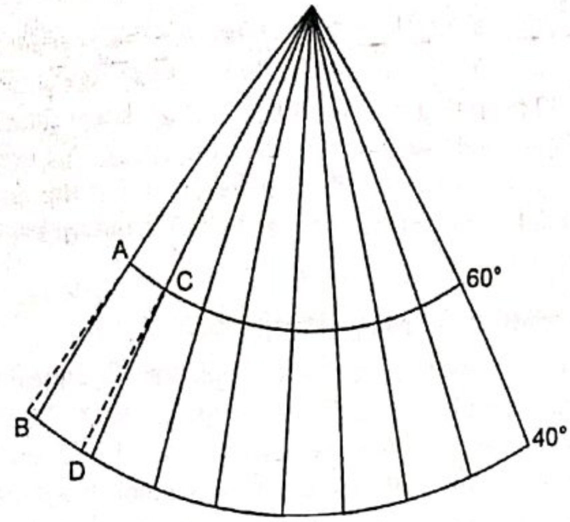

Fig. 266

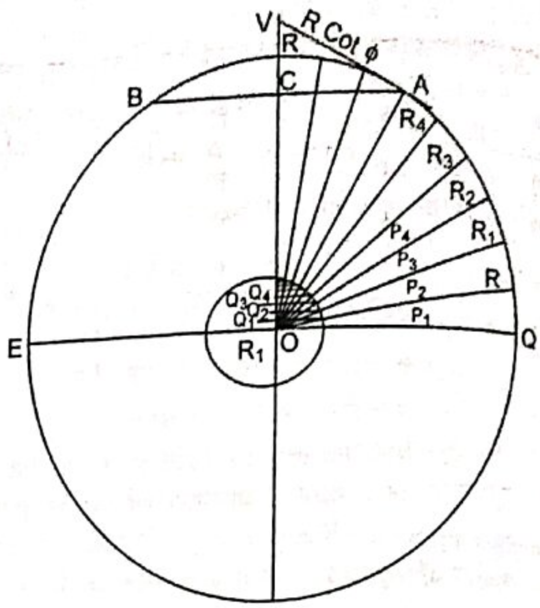

Fig. 267

projection, which is a special case of Bonne's projection when the equator is taken as the standard parallel. The Bonne's system is also used for the topographical sheets of small countries like France, Netherlands, Switzerland and Belgium.

EXAMPLE

Prepare graticule on Bonne's Projection for North America on the 1 : 25,000,000 scale with an interval of 10°.

Graphical Construction

According to the given scale the radius (R) of the reduced sphere

$$= \frac{250,000,000}{125,000,000} = 2''$$

From the centre O and with radius of 2″ describe a circle EBP (Fig. 267). Let AB be the standard parallel and VA be the tangent to the circle at A, which meets the prolonged axis OP at V. Then draw the central meridian VO and from the centre V draw the standard parallel with VA as radius, which cuts the central meridian at C. Starting from C, mark off the divisions on the central meridian equal to QR, when QOR = 10°, the given interval. With the centre V draw the parallels, passing through these points. From the centre O and with the radius QR, describe a semi-circle which intersects OR, OR_1, OR_2, OR_3, etc., at the points D_1, D_2, D_3, D_4, etc. respectively. From these points draw lines parallel to EQ to meet the axis OP at a_1, a_2, a_3, and a_4 etc. Then with the distances D_1a_1, D_2a_2, D_3a_3, D_4a_4, etc., mark-off the points on the respective parallels. Join the corresponding points to get the meridinal curves.

Trignometical Constructions

Let 60° N be the standard parallel; the radius of the standard parallel = R cot ϕ = 2 × cot 60° = 2 × 0.58 = 1.16″.

Y which denotes the true interval between the parallels

$$= \frac{2\pi R \times 10}{360} = 0.35''$$ where the interval is 10°.

X, the longitudinal distances along the parallels

$$= \frac{2\pi R \cos\phi \times 10}{360}.$$

With the help of the above formula, the longitudinal distances along each parallel have been tabulated as in Table 1A.

Now the graticule can be prepared as in Fig. 268. In drawing the meridinal curves, the use of French curves may be made. If the radii of parallels are too long to be drawn by an ordinary compass, the use of a beam compass may be made.

Polyconic Projection

The Polyconic Projection was developed by Ferdinand Hassler, an American cartographer and surveyor. In principle it represents the piling up of

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TABLE 1A

ϕ	cos ϕ	R	$2\pi R$	$2\pi R$ cos ϕ	X ($2\pi R$ cos ϕ/36)	R ($2\pi R/36$)
10	0.98	2″	12.6″	12.3″	0.34″	0.35″
20	0.94	2″	12.6″	11.8″	0.33″	0.35″
30	0.87	2″	12.6″	10.9″	0.30″	0.35″
40	0.77	2″	12.6″	9.7″	0.27″	0.35″
50	0.64	2″	12.6″	8.1″	0.22″	0.35″
60	0.50	2″	12.6″	6.3″	0.17″	0.35″
70	0.34	2″	12.6″	4.3″	0.12″	0.35″

as many hallow cones as the circles of latitude to which they closely correspond. Thus all the cones are tangent to the sphere along the corresponding parallels of latitudes, all of which subsequently become standard parallels. But these are not concentric circles as in the case of Simple Conic and Bonne's (See Fig. 269). VT, V_1T_1, V_2T_2, etc., the projected radii of the respective parallels, equal to the cotangent of latitude × radius of the reduced sphere, *i.e.*, R cot ϕ when ϕ is the lattitude and R, the radius of the reduced sphere. The central meridian and the parallels are divided in the same way as in Bonne's. Net unlike Bonne's meridians are smooth curves drawn by joining the points of division marked along the parallels.

This projection is neither conformal nor equal area. As we have seen, the scale is true only along the central meridian and parallels. The meridinal scale increases as we proceed away from the central meridian. The curvature of the meridians increases much rapidly beyond the first 30° of longitudes on either side of the central meridian. Hence various sheets of a country adjoining east and west cannot be correctly fitted together if they are drawn on this system with different central meridians. In fact, the projection is not suitable to a country which extends beyond 30° on each side of the central meridian.

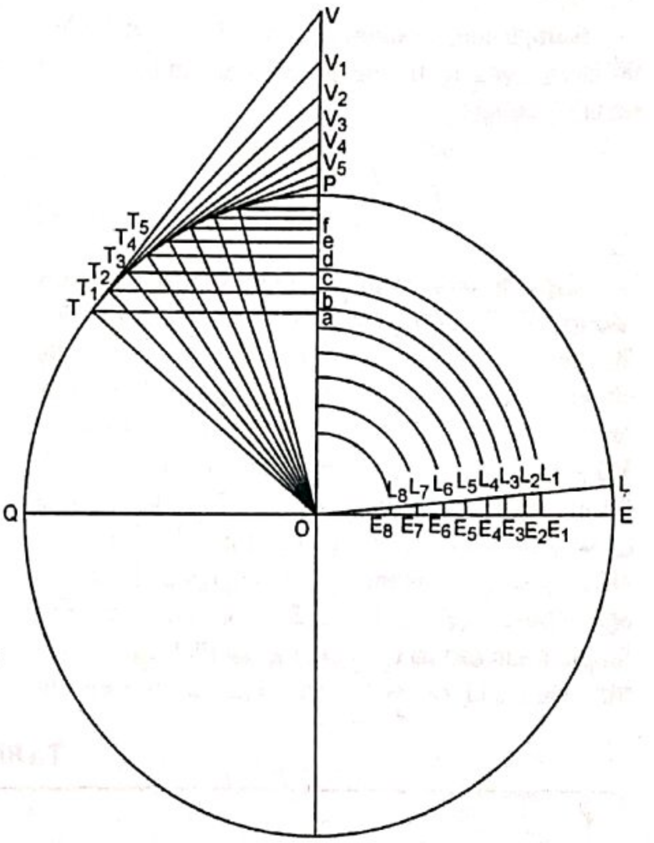

Fig. 269

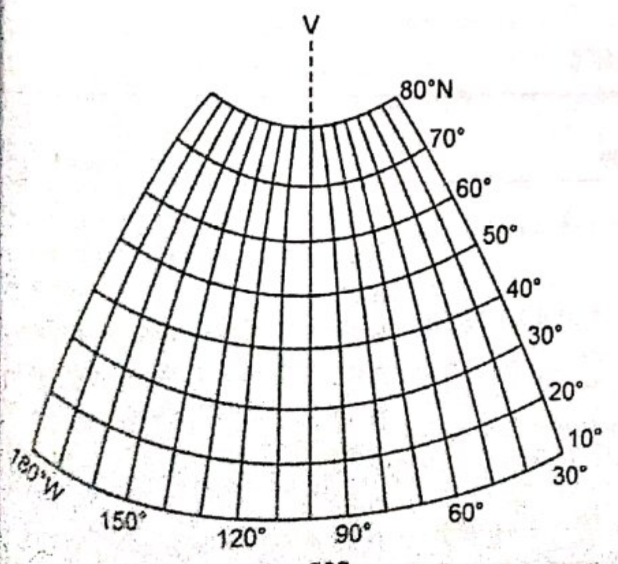

Fig. 268

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The shape is also distorted on the polar margins so it is restricted within 20° of the pole. The projection is, however, fit for a map of Europe. The area of a quadrangle of map prepared on this projection is not equal to the area of the corresponding quadrangle on the sphere, because the parallels are not concentric circles. The projection is, however, suitable for the topographic survey sheets prepared with their independent central meridians. It has been exten-sively used by American cartographers for various topographical survey sheets.

Graphical Construction for Europe on 1 : 62,500,000 Scale with 5° Interval :

Europe lies between 30° N–70° N and 10°W–60°E. According to the given scale radius of the reduced sphere,

$$R = \frac{250,000,000}{62,500,000} = 4''$$

Draw a circle from the centre *O* with 4" radius, and to OT, OT_1, OT_2, OT_3, OT_4 to represent 30°, 40°, 50°, 60° and 70° respectively. Draw tangents to the circle at the points T, T_1, T_3, and T_4, which meet the prolonged axis OP, at V, V_1, V_2, V_3, V_4. Thus VT, V_1T_1, V_2T_2, etc. are perpendiculars to the polar axis. From the centre *O* with *Ta*, *Tb*, T_2c, etc., as radii draw concentric arcs in the quadrant *POE*. The line *OL* drawn at 5° interesects these arcs at L_1, L_2, L_3, etc. Thus, L_1E_1, L_2E_2, L_3E_3, etc. are respective longitudinal distances along the parallels of 30°, 40°, 50°, etc., and *LE* is the true interval between the parallels 5° apart to be marked on the central meridian.

In order to complete the construction take a central meridian *VO*. From *O* mark-off the points *a*, *b*, *c*, *d*, etc. on it at a distance of *LE*. Through these points describe the circles of latitude with *VT*, V_1T_1, V_2T_2, etc., as radii. It may be noted here that the centres V_1, V_2, V_3, etc. will be moving along *OV* produced. Divide each parallel as in the Bonne's and draw smooth curves of the meridians by joining the respective points of division.

Trigonometrical Construction

Calculate radii (r) of the parallels on the projection with the formula, $r = R \cot \phi$, when ϕ is the latitude and R is the radius of the reduced sphere. The length of the parallels = $2\pi R \cos \phi$ as in the Bonne's; y the distance between two parallels $y = (2\pi R \times 5)/360$ when the interval is 5°, thus $y = 0.35''$; x the distance between two meridians at 5° interval measured along the parallels is equal to $\frac{2\pi R \cos \phi \times 5}{360}$. Thus the table 2 may be computed.

Draw the central meridian *VO* representing 25°E and from *O* mark-off *y* on it. Through the points *a*, *b*, *c*, *d*, *e*, etc., thus obtained, describe cricles of 35°, 40°, 45°, 50°, 55° etc., with corresponding *r*, given in the table. Then mark-off seven points at *x* distances along these to show 35° longitudes on each side of the central meridian. Joining these points by smooth curves, the meridians my be completed (Fig. 270).

TABLE 2

ϕ (Lat.)	$\cot \phi$	r ($R \cot \phi$)	$\cos \phi$	x ($y \cos \phi$)
30	1.73	2.92"	0.87	0.30"
35	1.43	5.72"	0.82	0.29"
40	1.19	4.70"	0.77	0.27"
45	1.00	4.00"	0.71	0.25"
50	0.84	3.30"	0.64	0.22"
55	0.70	2.80"	0.57	0.20"
60	0.58	2.32"	0.50	0.18"
65	0.47	1.88"	0.42	0.15"
70	0.36	1.44"	0.34	0.12"

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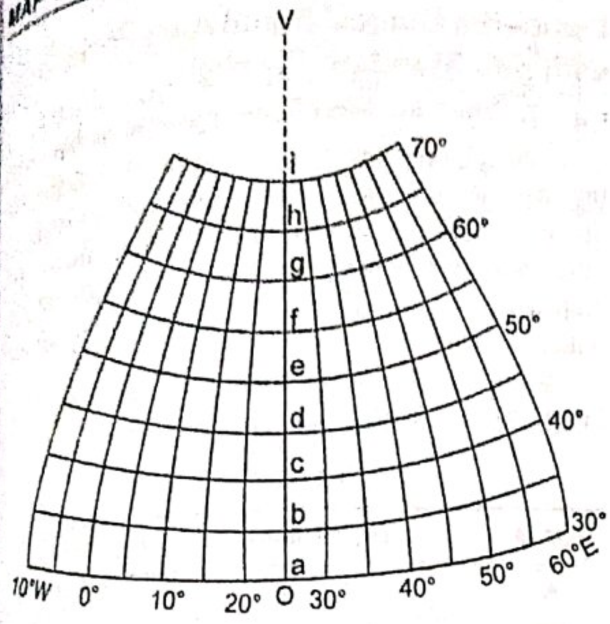

Fig. 270

Conical Equal Area Projection with Standard Parallel

This projection is also called 'Lambert's Conical Equal-Area Projection.

It is a modified form of simple conical projection with one standard parallel. The modification is made to make it an equal area projection. Like the simple conic projection, in Lambert's Conical Equal-Area Projection too, the meridians are all radial straight lines, placed at equal angular intervals and the parallels are all concentric arcs. The scale is correct along all the other parallels.

But unlike the simple conic projection, in this projection the meridional scale is not correct. The exaggeration of scale along the parallels is made good by a proportionate minimisation of scale along the meridians.

Thus the parallels are unequally spaced from each other.

The main problem is then to find out the lengths of the radii with which the different parallel (other than the standard one) are to be drawn on the projection. The rest of the construction is similar to that of the simple conic projection.

The following formula, derived mathematically, may be used to find out the lengths of the radii for drawing the concentric arcs to represent the various parallels :

$$r_1 = R\sqrt{\cot 2\phi_2 + 2 - \frac{2\sin\phi_1}{\sin\phi_0}}$$

where r_1 is the required radius for drawing the parallel of ϕ_1 latitude.

ϕ_0 is the standard parallel and R is the radius of the reduced earth according to the given scale.

Select the standard parallel which will be the middle parallel to the latitudinal zone for which the graticule is to be drawn.

Find out the radius with which the standard parallel is to be drawn on the projection. It may be found out mathematically, or graphically, the method being similar to that in the case of simple conic projection with one standard parallel.

Construction

Draw a vertical line to represent the central meridian. Mark a suitable point along this line to represent its intersection with the standard parallel. The radius of the standard parallel on the projection is already known, hence the common centre with which the parallels are to be drawn can easily be marked. With this common centre marked along the central meridian and with radii calculated with the help of the above formula draw the different parallels as arcs of concentric circles. The standard parallel should be truly divided and intercepts along it for drawing the meridians at the given interval may be

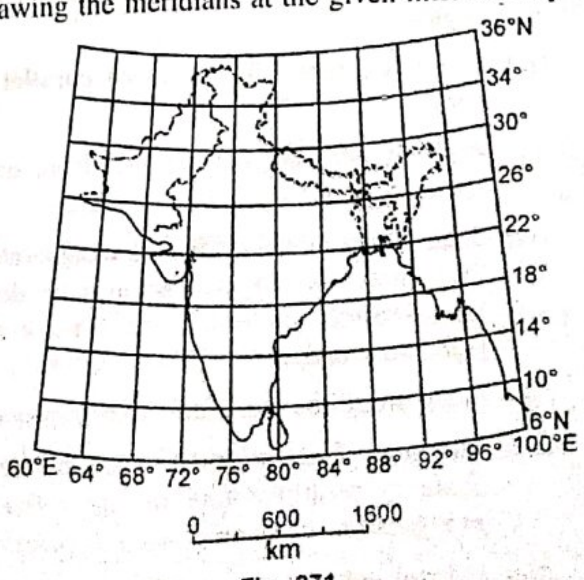

Fig. 271

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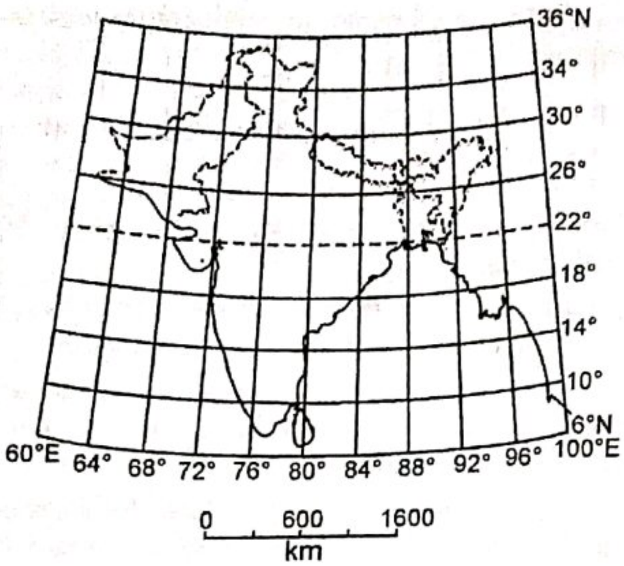

Fig. 272

found out graphically or mathematically as in the case of simple conic projection with one standard parallel. Then the required meridians may be drawn as radial straight lines from the common centre passing through the points of intersection already marked along the standard parallel. Thus the graticule is complete (Figs. 271 & 272).

Properties

(*i*) The parallels are all arcs of concentric circles.

(*ii*) The meridians are all radial straight lines placed at equal angular intervals.

(*iii*) The parallels intersect the meridians at right angles.

(*iv*) The scale along the standard parallel is corect.

(*v*) Parallels are unequally spaced from each other.

(*vi*) Scale along other parallels is exaggerated. The amount of exaggeration in scale along the parallels increases away from the standard parallel.

(*vii*) Scale along the meridians is minimised.

(*viii*) The ratio of minimisation of the meridinal scale is proportionate to the ratio of exaggeration of scale along the parallels.

(*ix*) It is an equal area projection.

Lambert's Conical Equal Area Projection with one Standard Parallel

Case I. When the Meridians converge at the Pole

Let r_0 be the radius of the standard parallel on the map, ϕ_0 and X_0 be the latitude and co-latitude of the standard parallel respectively, ϕ_1, ϕ_2, ... be other parallels whose corresponding radii on the map are r_1, r_2, ... and let R be the radius of the reduced earth.

For a map of India let the extend be 6°N–38°N (Table 3) and 60°E–100E and interval 4°.

TABLE 3

Lat. ϕ	Case I (inch)	Case II (inch)
6°	20.17670	34.3875
10°	19.38365	33.5375
14°	18.56645	32.6750
18°	17.72510	31.8125
22°	16.86250	30.9386
26°	15.97915	30.0625
30°	15.07750	29.1750
34°	14.15775	28.3375
38°	13.21995	27.5000

Let the standard parallel be 22°N and the scale be 1 : 20,000,000.

The formula used are

$$r_0 = 2R \tan \frac{x_0}{2}$$

and

$$r_1 = 2R \sec \frac{x_0}{2} \sin \frac{x_0}{2}$$

(x_1 is the co-latitude of the parallel ϕ_1)

(Radius of the globe is supposed to be 250,000,000").

Intercept on the standard parallel

$$= \frac{2\pi R \cos 22°}{360°} \times 4° = 0.80914 \text{ inch}$$

Parallels are concentric circles and meridians are staight lines converging at the pole.

Case II. When the Meridians converge beyond the Pole

Let r_0 be the radius of the standard parallel on the map, ϕ_0 be the latitude of the standard parallel,

$\phi_1, \phi_2, \ldots$ be other parallels whose corresponding radii on the map are $r_1, r_2 \ldots$ and let R be the radius of the reduced earth.

For a map of India let the extent be 6°N–38°N (Table 3) and 60°E–100°E and interval 4° and standard parallel 22°N; let the scale be 1 : 20,000,000.

The formula used are

$$r_0 = R \cot \phi_0$$

and $$r_1 = \sqrt{\cot 2\phi_0 + 2 - \frac{2 \operatorname{cosec} \phi_1}{\sin \phi_1}}$$

(Radius of the globe is supposed to be 250,000,000").

Intercept on the standard parallels

$$= \frac{2\pi R \cos 22^\circ}{360^\circ} \times 4^\circ = 0.80914 \text{ inch}$$

Parallels are concentric circles and meridians are staight lines converging at the pole.

*Fig. 272(*a*) represents a map of India on the Conical Equal Area Projection with one standard parallel, a projection which was devised by Lambert in the year 1772; while Fig. 272(*b*) is drawn on the same projection which contains new suggestions.

In the first case the apex of the cone coincides with the pole whereas in the second case the pole is represented by an arc. Again in the former all the meridians converge at the pole, while in the latter they converge beyond the pole (*c.f.* Table 3A).

Zenithal Projection

When the nets are obtained by projecting the lines of latitudes and longitudes on a surface, which is tangent to the globe at a point, they are known as Zenithal or Azimuthal projections. The word Azimuthal carries the full connotation of the nets in the sense that they show the correct bearings or azimuths of all the points from the centre of the maps. This is an unique property possessed singularly by the projections of the group. Further they can be used for mapping any part of the world and for any purpose, one desires, because a plain can be tangent to the globe at infinite points. It has two broad divisions.

1. Perspective zenithal projections.
2. Non-perspective zenithal projections.

Perspective Zenithal Projections

Stereographic Polar Zenithal Projection

This is one of the perspective zenithal projections. If you place the light at one pole and the screen as tangent plane at the other pole, a shadowed picture of the hemisphere will be obtained. This system is known as the Stereographic Polar Zenithal Projection. Like other zenithal projections, meridians are straight lines and the parallels are concentric rapidly from the centre of the map; so the projection gives a very distorted view as contrary to the orthographic projection. The meridional distance also increase towards the equator in the same proportion; hence correct shape of smaller areas is maintained.

TABLE 3A

1st Case	*2nd Case*
1. Meridional scale is exaggerated polewards and is minimised equatorwards of the standard parallel.	1. The meridional scale is minimised on both sides of the standard parallel.
2. The scale along the parallel is minimised polewards and exaggerated equatorwards of the standard parallel.	2. The scale along all the parallels excepting the standard parallel is exaggerated.
3. The scale along the meridians is inversely proportional to the scale along the parallel to preserve the equal area property.	3. The same.

*Raisz : Erwin, *General Cartography*, (1948), p. 75.

The direction of all lines from the centre is also true. Therefore the projection is both orthomorphic and azimuthal. This is commonly used for the map of the world in hemispheres by cato-graphers.

Graphical Construction

In the diagram *TG* is the tangent plane touching the sphere at *P* and light is thrown from the opposite end *L* (Fig. 273). The ray *LM* passes through the point *A* which lies at an angular distance of, say, 60° from the equator. Then *PM* will be radius of 60°N parallel on the projection because the point *A* is projected to *M* on the tangent plane. Similarly, the equator will be projected to *R* and *PR* will become the radius for drawing the equator. Likewise, the radii for other parallels may be found. The parallels will all be drawn as concentric circles with their projected radii. The meridians will be drawn as radial straight lines placed at true angular intervals.

Trigonometrical Construction

In Fig. 273, in the right-angled ΔLPM, the perpendicular

$$PM = LP \tan \angle MLP$$

$$= 2R \tan \frac{\angle POA}{2}$$

because the angle at the centre is double the angle at the circumference when subtended by the same arc.

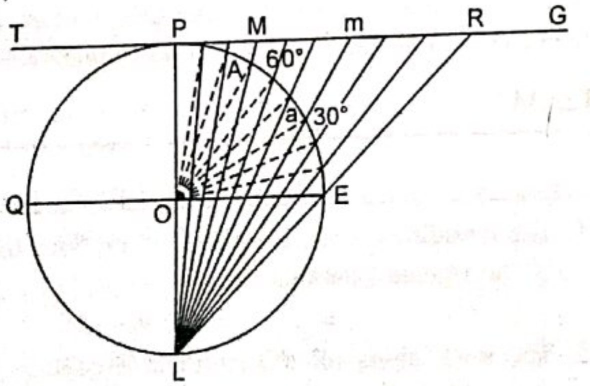

Fig. 273

Therefore *r*, the radius of the 60°N parallel = 2*R* . 1/2 co-latitude = 2*R* tan . *Z*/2, when Z is co-latitude and *R*, radius of the reduced sphere. With the above formula the radii of all the parallels may be calculated as tabled (4) and the meridians may be drawn at the given interval from *p* as radial straight lines (Fig. 274).

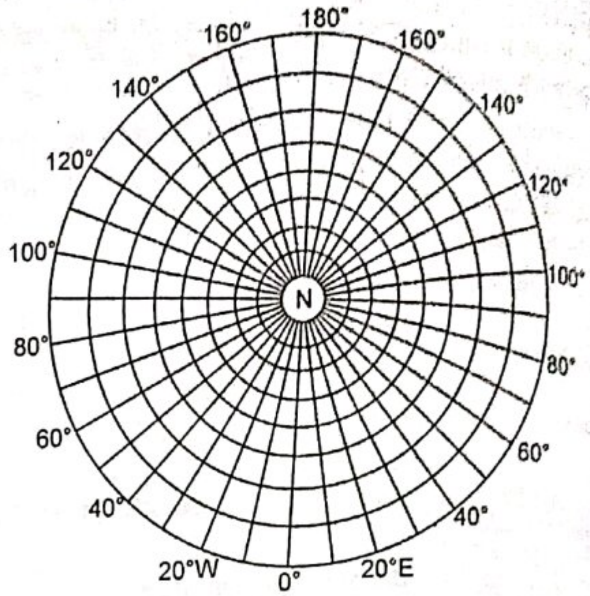

Fig. 274

Table 4 showing radii of parallels for one hemisphere on scale 1 : 250,000,000.

TABLE 4

2R	Latitude	Z/2 co-lat/2	tan Z/2	r
4″	10	40	0.84	3.36″
	20	35	0.70	2.80″
	30	30	0.58	2.32″
	40	25	0.47	1.88″
	50	20	0.36	1.44″
	60	15	0.27	1.08″
	70	10	0.18	0.72″
	80	5	0.09	0.36″

Gnomonic Polar Zenithal Projection

In this projection the source of light is supposed to be at the centre of the sphere and the tangent touches either of the poles. Like the stereographic, it is also a perspective projection. It is impossible to draw the map of one hemisphere on this system because the equator becomes infinite (Fig. 275). The scale increases very rapidly towards the margins of the map, and therefore the projection is suited only for small areas round the pole. There is one special merit in it due to which it is mostly used in charts for nvigation : all great circles appear

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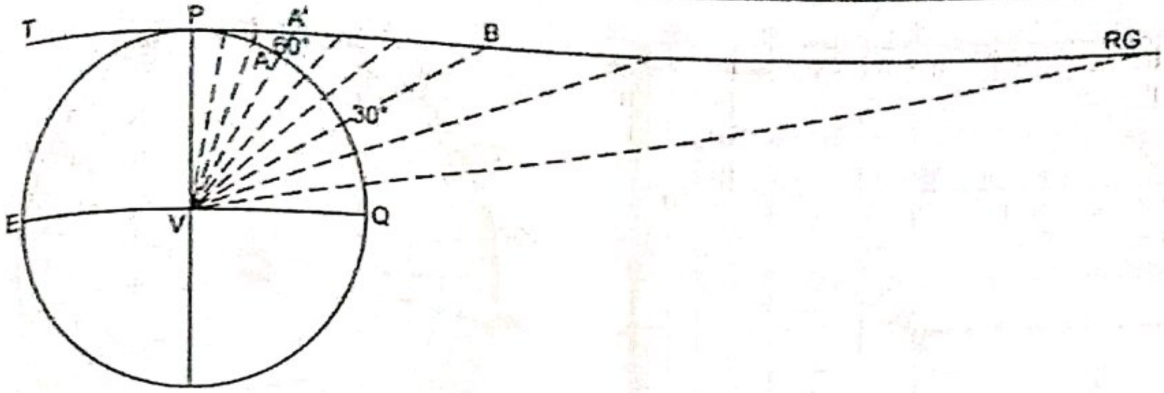

Fig. 275

as straight lines because their planes pass through the centre of the sphere, where lies the source of light for this perspective projection. That is, if you want to find the shortest distance between two points on the map you need only join them by a straight line.

Graphical Construction

In Fig. 275, *P* is the north pole at which the plant *TG*, is tangent; *V* is the source of light lying at the centre of the sphere. The rays *VA*, *VB*, etc., pass through the latitudes of 60°, 40°, etc., and meet the tangent plant at *A'*, *B*, etc. Thus *PA'*, *PB*, etc., are the projected radii of the corresponding parallels.

Trignometrically, the values of *PA'*, *PB*, etc., may be easily calculated as follows :

In the right-angled $\Delta A'PV$,

$$\frac{A'P}{PV} = \tan \angle A'VP$$

$$A'P = \angle PV \tan \text{co-latitude.}$$

Therefore, $r = R$ tan co-latitude, when r is the projected radius of the parallel and R denotes radius of the reduce sphere.

Now, with this formula, the projected radii of all the circle of latitude on the projection may be calculated as in Table 5.

With the help of the Table 5 the graticule for S. Australia and New Zealand may be prepared as in Fig. 276.

TABLE 5. Radii of Parallels for Southern Hemisphere extending from 20°S to 80°S on 1 : 250,000,000 scale

R	ϕ = *Latitude*	*Co-lat.*	*tan co-lat.*	*r*
2"	20	70	2.75	5.50"
	30	60	1.73	3.46"
	40	50	1.19	2.38"
	50	40	0.84	1.68"
	60	30	0.58	1.16"
	70	20	0.36	0.72"
	80	10	0.18	0.36"

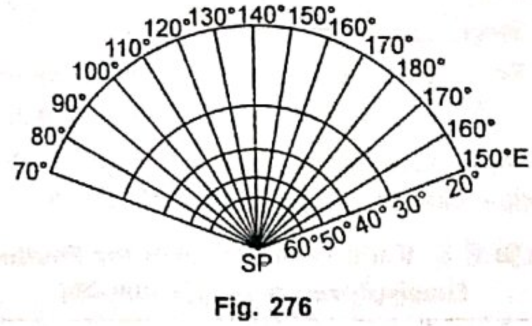

Fig. 276

Orthographic Polar Zenithal Projection

In this case light is thrown from a point at infinity, on the tangent plane touching the sphere at the pole. The rays of light passing through the latitude are parallel to each other (Fig. 277).

In Fig. 277 the rays from infinity pass through the latitudes and the point *L* is projected to *B*, the *LB* being perpendicular to *TG*, the tangent plane, at *B*. Thus *PB* is the radius of the parallel *L* (30°). Similarly the radii of other parallels may be found by dropping perpendiculars to the tangent plane from the corresponding points of latitudes, through which the rays are supposed to pass.

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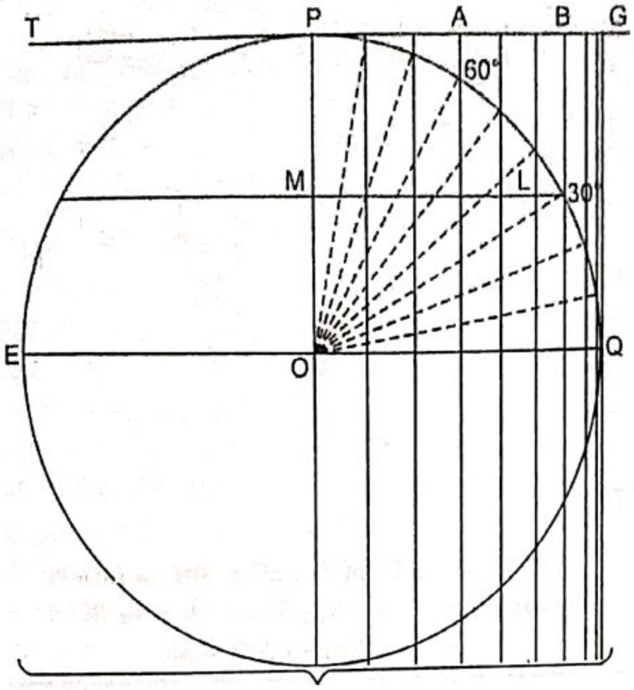

Fig. 277

Mathematically, too, the value of *PB*, *PA*, etc., may be calculated. In the right-angled ΔLMO,

$$\frac{LM}{OL} = \sin MOL = \cos \angle OLM$$

or $LM = OL \sin \angle MOL = OL \cos \angle OLM$

$= R$ sin co-latitude $= R$ cos latitudes,

where R is the reduced sphere and $\angle OLM = \angle LOQ$ = Latitude.

Since $PB = LM$

So $PB = R$ sin, co-lat. or $R \cdot$cos latitude

$r = R$ sin co-lat. or R cos latitude

Now, with this formula the radii of all the parallels may be calculated as Table 6 below.

TABLE 6. Radii of all Parallels for Southern Hemisphere on 1 : 250,000,000

R	*Latitude*	*Co-lat.*	*sin co-lat.*	*r*
2″	10	80	0.98	1.96″
	20	70	0.94	1.88″
	30	60	0.87	1.74″
	40	50	0.77	1.54″
	50	40	0.64	1.28″
	60	30	0.50	1.00″
	70	20	0.34	0.68″
	80	10	0.17	0.34″

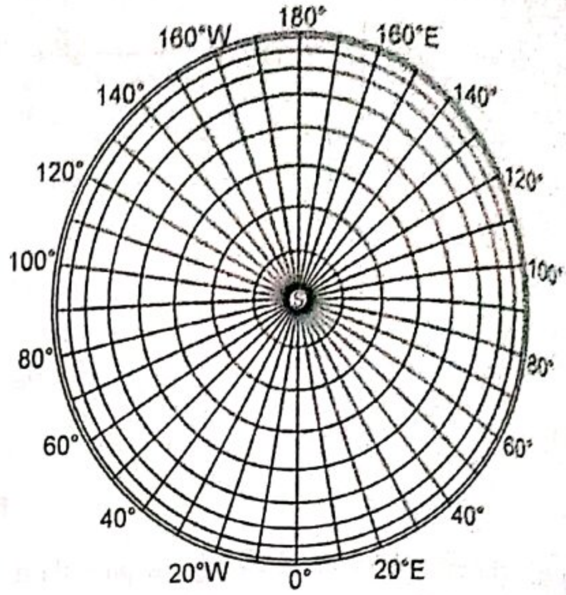

Fig. 278

The graticule may be prepared in the same way as in the other Polar Zenithal Projections (Fig. 278). The projection is useful to astronomers who can see the position of heavenly bodies every time on such orthographic maps showing these.

Non-Perspective Zenithal Projections

Polar Zenithal Equal Area Projection

This graticule was designed by J.H. Lambert in 1772. It has become popular in recent years. Like other Zenithal Projections, in this system, too, there meridians are straight lines, drawn at their true angular distances radiating from the pole and the parallels are concentric circles. The circles of latitude become closer away from the pole. Their spacing is so adjusted as to make it an equal area projection. This is most commonly used for polar areas in atlases : but on this projection the world may be represented in hemispheres with the pole areas as centre.

From the diagram (Fig. 279) it is evident that *EP*, the radius of the circle *EQA*, which represents the projected equator on the map of one hemisphere, is equal to $\sqrt{2}$ times the radius of reduced sphere *EPQ*. The area of one hemisphere $= 2\pi R^2$ and the area of a circle is πr^2. Thus the area of the circle $EQA = \pi(\sqrt{}R)^2 = 2\pi R^2$ which represents the surface of one hemisphere. Similarly, it may be proved that the area of every latitudinal zone on the map will be equal to the corresponding one on the globe.

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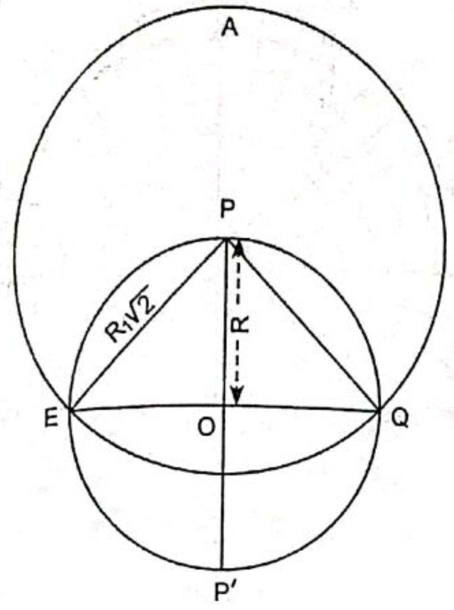

Fig. 279

Graphical Construction

Let the circle *ENQ* be drawn with centre *O* and with radius equal to that of the reduced sphere on a given scale. Join *NO* and produce it to *S*. Draw *OL* at a distance of ϕ, denoting latitude. Join *LN*, which is the required radius for the parallel ϕ. Similarly *NL* etc., are the radii of other parallels drawn at some given interval. *NE* will be the radius of the equator (Fig. 280).

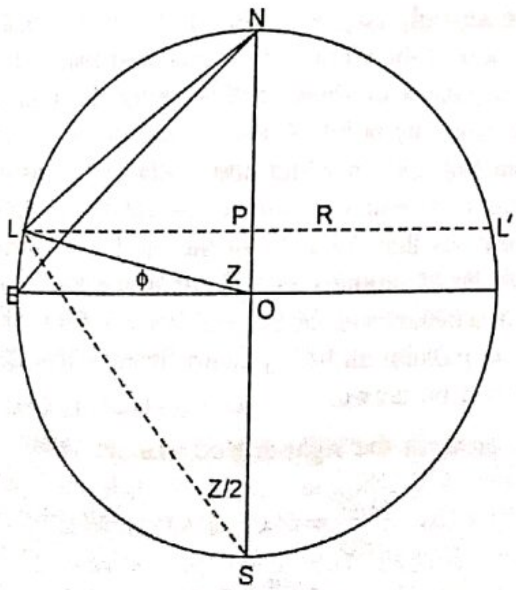

Fig. 280

The value of *LN* may be calculated. Join *L S*. In the right-angled $\Delta NLS'$,

$LN = NS' \sin \angle NSL = 2R \sin 1/2$ co-latitude,

because in the angle $LSN = 1/2 \angle LON$ the co-latitude (*Z*) and $NS = 2R$

$$r = 2R \sin \frac{Z}{2}.$$

With the help of the above formula, the values of the radii in inches of the parallels at 10° interval on 1 : 50,000,000, scale has been given in the Table 7.

TABLE 7

R	*Latitude*	*1/2 Z*	*sin 1/2 Z*	*r*
2″	10	40	0.643	6.43″
	20	35	0.574	5.74″
	30	30	0.500	5.05″
	40	25	0.423	4.23″
	50	20	0.342	3.42″
	60	15	0.259	2.59″
	70	10	0.174	1.74″
	80	5	0.084	0.84″

A outline map of Asia may be prepared on this projection (Fig. 281). The equatorial case of this projection is generally used for hemisphericals maps but its construction is beyond the scope of our present study.

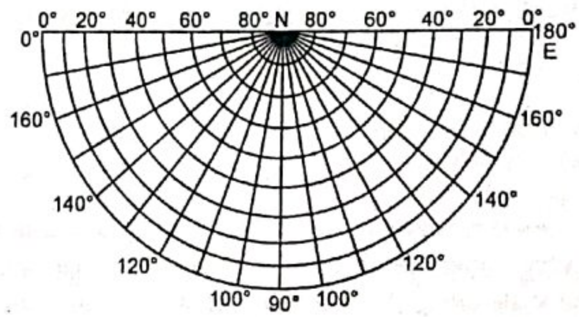

Fig. 281

Polar Zenithal Equidistant Projection

It has derived its name from the fact that the parallels are equidistant on the graticule. Unlike other polar Zenithal projections, this is an arbitrary projection, and not a perspective projection because the parallels cannot be projected equidistant in any case of view-point (*V*). In this case the parallels are placed at their true distances; as such, their interval can be easily calculated by the formula,

$D = \frac{2\pi R \times d}{360°}$ where *D* is the distance, *R* is the radius

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of the reduced sphere and d is the given interval in degrees.

Graphical Construction

Draw the circle EPQ, from the centre O and with the radius of the reduced sphere. Make OR to show the given interval, say, 10° (Fig. 282). QR is the true distance at which the parallels may be spaced. Thus from the centre P, the circles of latitude may be described, making the distances between two parallel equal to QR. Meridians will be drawn by protractor from P at the given interval as in the case of other Polar Zenithal projections. (See Fig. 282 and 283).

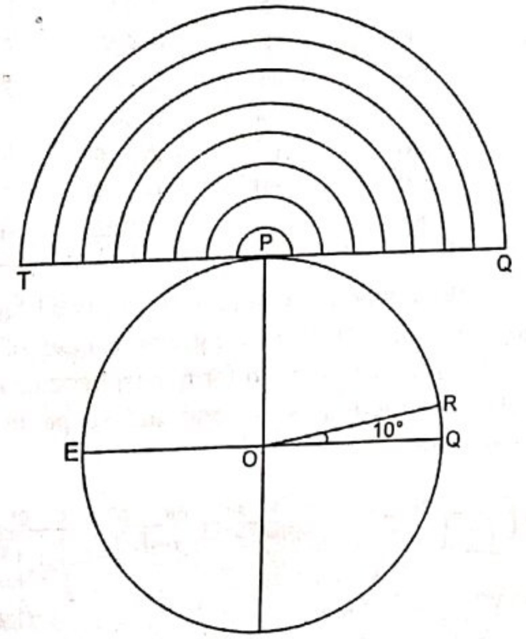

Fig. 282

As it is very easy to construct this projection, it is very commonly used for the map of polar area. The scale along the parallels increases greatly away from the map so the projection may be fairly good only for small areas around the pole not exceeding 30° in latitudinal extent. On a map drawn on this system, the distance and bearing of any point from the pole are correct.

Stereographic Normal Zenithal Projection

In this projection the tangent plane touches the globe at any point along the equator and the source of light is at a diametrically opposite point. Like all other normal zenithal projections, in stereographic zenithal projection, the central meridian and the equator both are straight lines cutting each other at

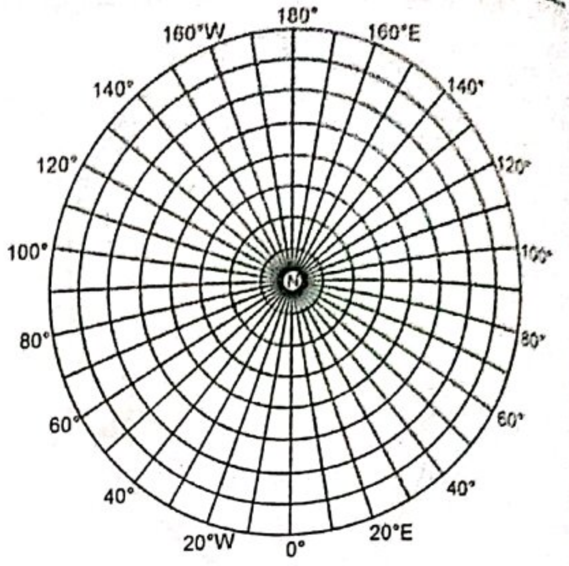

Fig. 283

right angles. The globe is represented in hemispheres on this projection. But in all the stereographic projections all angles on the sphere are reproduced equally in the projection and all the circular arcs are projected as circular arcs except the equator and the central meridian. The projection is, therefore, orthomorphic.

Construction

The polar axis and the equtorial axis can be represented by two straight lines intersecting each other all right angles but the problem is to find out the radii of the circles of latitude and longitude and the distances of their centres along the respective axes from the point of intersection of the two axes. From Fig. 284 in which the circle is drawn with a radius ($2R$) equal to double the radius of globe, it is obvious that the radii of the circles representing parallels of latitude are $2R \cot \phi$ and their centres are at a distance of $2R \operatorname{cosec} \phi$ from O. CB is tangent to the parallel and C is centre from which the arc AB will be drawn.

Now, in the right-angled triangle CBO,

$$\frac{CB}{BO} = \cot \phi \text{ or } CB = OB \cot \phi$$

$$= 2R \cot \phi$$

Similarly, $$\frac{OC}{OB} = \operatorname{cosec} \phi, \text{ or } OC = OB \operatorname{cosec} \phi$$

$$= 2R \operatorname{cosec} \phi.$$

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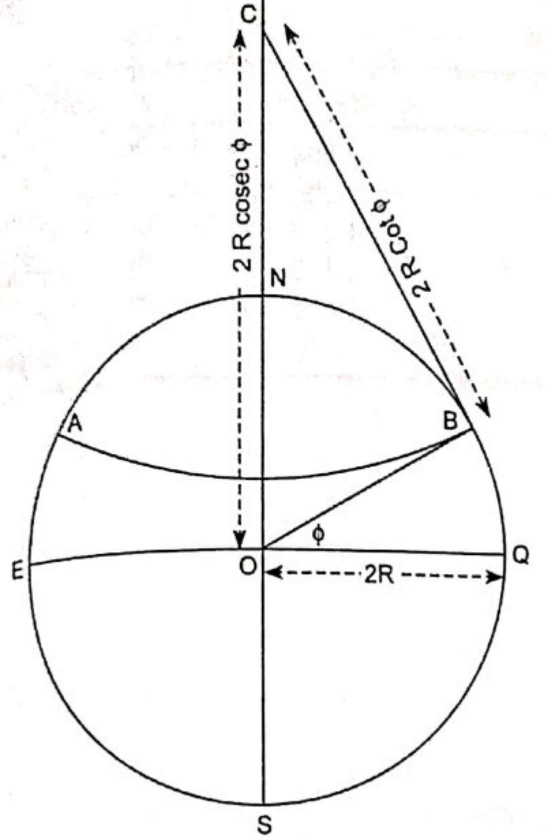

Fig. 284

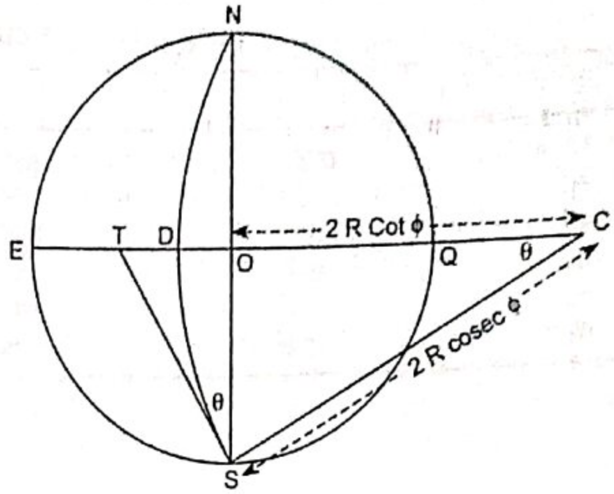

Fig. 285

In Fig. 285, C is the centre of longitudinal arc NDS and CS is its radius and ST is tangent to the longitude of ϕ at S. The $\angle TSO = \angle OCS = \phi$.

Now, in the right-angled ΔCOS,

$$\frac{OC}{OS} = \cot\phi \text{ or } OC = OS \cot\phi$$

$$= 2R \cot\phi$$

Again, $$\frac{CS}{OS} = \text{cosec}\,\phi; \text{ or } CS = OS \text{ cosec}\,\phi$$

$$= 2R \text{ cosec}\,\phi.$$

Thus, the centre of longitudinal circles will be at a distance of $2R \cot\phi$ from the point O and the length of their radii will be $2R$ cosec ϕ.

Graphical Construction

If R be the radius of the globe according to the given scale, then draw a circle $NESEQ$ with radius $2R$ to represent one hemisphere (Fig. 285). Let NS and EQ, intersecting each other at right angles, represent the central meridian and the equator respectively.

Suppose the parallels and meridians are to be drawn at intervals of ϕ degrees.

Construction of Parallels

Draw the radius OA making the angles EQA equal to ϕ. Then draw AC perpendicular to OA meeting the central meridian SN, extended at C.

With centre C and radius CA draw an arc of a circle meeting the circumference of the circle $NESQ$ at A and B. The arc AB is required parallel of ϕ. Similarly changing ϕ to 2ϕ, 3ϕ, 4ϕ, and so on, the subsequent parallels, all being at intervals of ϕ degree, can easily be drawn. The method of construction will remain the same. In a similar way the parallels on the other side of equator may also be completed as the parallels on either side of the equator are symmetrically arranged.

Construction of meridians

From S draw a line ST making the angle NST equal to ϕ degree. Then draw SC perpendicular to TS meeting the equator EQ (produced if necessary) at C.

With centre C and radius CS draw the arc SDN. Now this arc SDN is the required meridian lying at an angular distance of ϕ degree from the central meridian. Similarly, changing ϕ to 2ϕ, 3ϕ, 4ϕ, ..., and so on, the subsequent meridians, all lying at intervals of ϕ degree, can be drawn. The method of construction will remain the same. In a similar way the meridians lying on the other side of the central meridian can also be completed.

EXAMPLE

Prepared a graticule for the Indian Ocean on the scale of 1 : 25,000,000, at an interval of 15° (*cf.* Table 8), and $2R = 4''$.

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TABLE 8

φ	cot φ	R cot φ	cosec φ	2R cosec φ
15	3.732	14.928″	3.864	15.456″
30	1.732	6.928″	2.000	8.000″
45	1.000	4.000″	1.414	5.656″
60	0.577	2.308″	1.155	4.620″
75	0.268	1.072″	1.035	4.140″
90	0.000	0.000″	1.000	4.000″

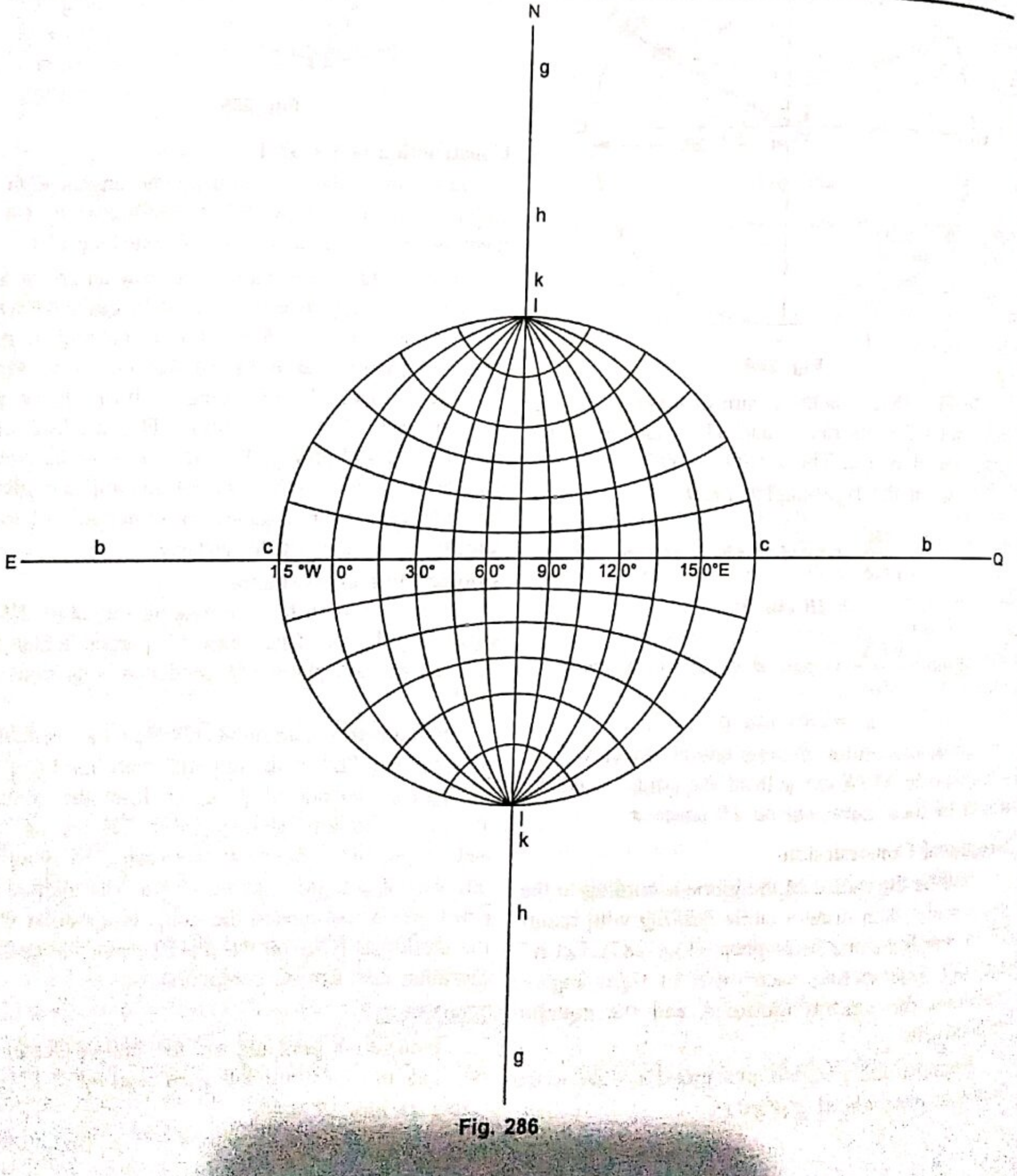

Fig. 286

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Let EQ and NS be the two axis of the central meridian and the equator intersecting at O. From g mark off the centre of parallels and meridians along NS and EQ respectively at the respective distances taken from the table. The centres of parallel 15°, 30°, 45°, 60° and 75° will be f, g, h, k, l respectively and those for the meridians of 15°, 30°, 45°, 60° and 75° will be a, b, c, d, e, respectively. The bounding meridians will be drawn as a circle with radius $2R$ = 4" from O, and the points where it will intersect NS will be representing the poles. Then with the centres thus marked and the radii equal to $2R \cot \phi$ the parallels will be drawn, and the meridians will drawn with the radii equal to $2R \operatorname{cosec} \phi$. The value of $2R \cot \phi$ and $2R \operatorname{cosec} \phi$ for the required latitudes and longitudes is entered in the table. In this way the required graticule will be obtained (Fig. 286).

Natural Cylindrical Projection

This is a perspective cylindrical projection. When a cylinder is wrapped round the globe so as to touch it along the equator, and light is placed at its centre, the true cylindrical projection is obtained. The exaggeration of the parallel scale as well as meridional scale would be very greatly increasing away from the equator. On this projection is scale would be true only along the equator. The poles cannot be shown because their distances from the equator become infinite. This projection serves no useful purpose as some non-perspective and modified projections do. For instance, the simple cylindrical, cylindrical equal area, Mercator's and Gall's Projection, etc., are generally used for various maps of the tropical regions or the world as a whole.

Simple Cylindrical Projection

The Simple Cylindrical Projection is also known as the Equidistant Cylindrical Projection because in this projection both the parallels and meridians are equidistant. They are drawn as straight lines, cutting one another at right angles. As the distance between the parallels and meridians is the same, the whole network represents a series of equal squares. All the parallels are equal to the equator ($2\pi R$) and all the meridians are half of the equator in length. The scale along the equator is true. The meridian sacle, *i.e.*, the north-south scale is also correct everywhere on the map because the parallels are drawn at their true distances. But the latitudinal scale increases away from the equator; this leads to great distortion in shape and exaggteration of area in high latitudes. Therefore the projection is neither orthomorphic nor equal area.

Construction

Prepare a graticule for the world map on the scale of 1 : 250,000,000 at 10° interval.

On the given scale R = 1" and the length of the

$$\text{equator} = \frac{2 \times 22 \times 1}{7} = 6.3''.$$

The true distance at which the parallels and meridians will be spaced is equal to $\frac{6.3 \times 10}{360} = 0.17''$. This may also be found out graphically. Draw a circle from the centre O with 1 radius (Fig. 287). Make the angle ROE = 10°. Now ER is the true distance at 10° interval between the parallels and the meridians. Let the equator be represented by

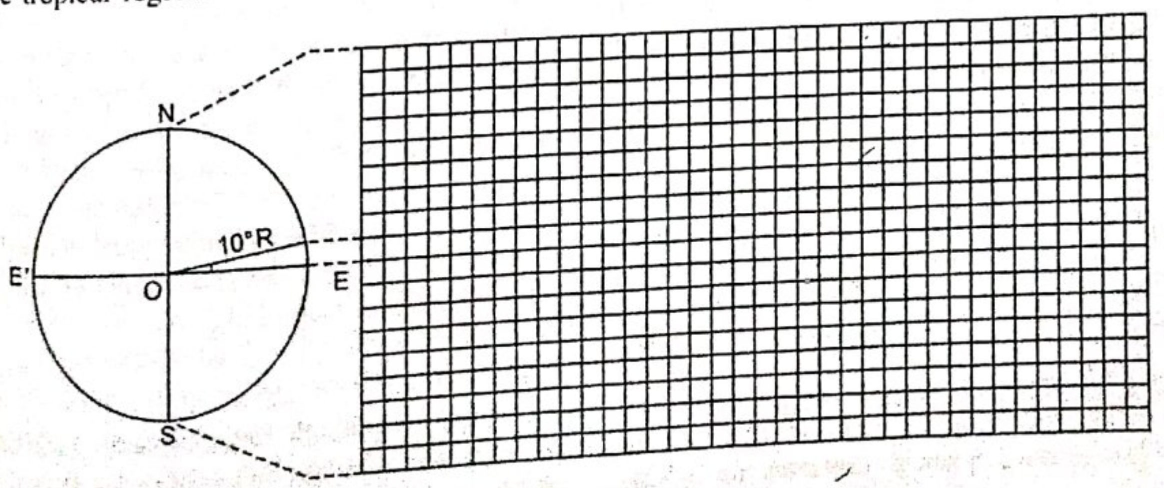

Fig. 287

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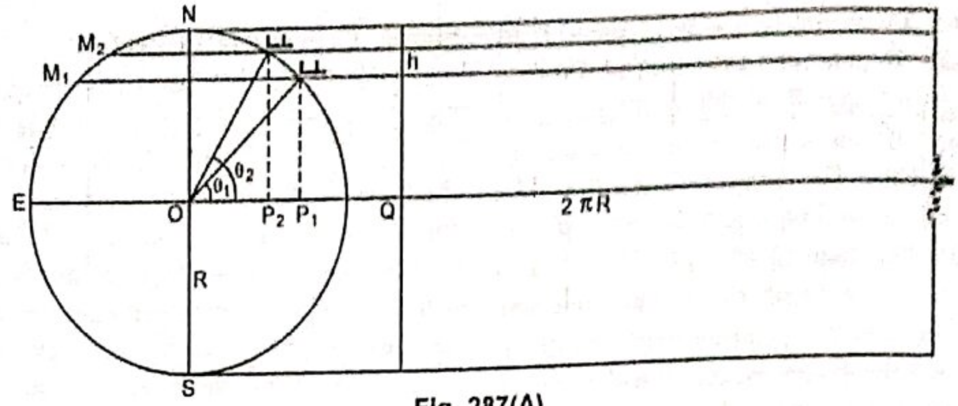

Fig. 287(A)

EQ. From its middle point *O* draw the central meridian *NS* at right angles to it. Make *NS* equal to half of *EQ*. Divide *EQ* and *NS* into 36 and 18 equal divisions respectively. Each division will be equal to 0.17″ or *ER*. From the points of division marked along them, draw lines parallel to the central meridian and the equator respectively so as to obtain other meridians and parallels. In this way the graticule may be completed as in Fig. 287.

Cylindrical Equal Area Projection

The Cylindrical Equal Area Projection, one of the Lambert's, has been derived by projecting the surface of the globe with parallel rays on a cylinder, touching it at the equator (Fig. 256). The circles of latitude and longitude both are projected as straight lines intersecting one another at right angles. The area between two parallel is made equal to the corresponding surface on the sphere at the cost of great distortion in shape towards higher latitudes; this is why it is an equal area projection.

In Fig. 256, the zone *QEA′V″* of the cylinder represents the projection of the zone *ABQE* on the sphere. The area of the zone *AEQB* on the sphere = the area of the zone *A′EQB′* of the cylinder = $2\pi Rh$, when *h* is the vertical interval between *EQ* and *AB*, the limiting parallels. The value of *h* may be mathematically calculated. Let *h* be represented by *OM* abd *OA* be the radius of the redueced sphere *NES* with is touched by a hollow cylinder along the equator *EQ*. *AB* is the parallel drawn at $\phi°$ distance.

Then $\dfrac{OM}{OA} = \dfrac{h}{R} = \sin\phi$ because $\angle EOA = \angle OAM$.

or, $h = R\sin\phi$, when ϕ represents the latitude.

With the help of the above formula the intervals between the equator and all other parallels may be easily calculated. As seen in Fig. 256, the equator is truly projected when the cylinder is cut open along the line *NS* and all other parallels including the pole have the same length. The meridians will be 2*R* in length and will be equally spaced. Thus, east-west extensions in area is made good by north-south compression. Therefore the area is represented accurately on this projection but the shape is distorted. The projection almost retains the quality of orthomorphism only near the equator where the amount of distortion is the least.

The projection is sometimes used for world maps to show distribution of commodities, etc. But it is suitable for the maps of the equatorial region in which both the shape and area are more or less, correctly represented.

Graphical Construction

Draw a circle from the centre *O* with the radius of the reduced sphere as in Fig. 287A. Produce *OE* to *Q*, making *EQ* equal to the true length of the equator ($2\pi R$). Draw the central meridian *NS* equal to *PP′* from the middle of *EQ*, at right angles to it. Draw *OA*, *OB*, *OB′* etc. at the given interval. From the point *A*, *B*, *B′* etc., draw lines parallel to *EQ*. The other meridians will be drawn at equal intervals and parallel to the central meridian. Thus *TGLM* is the required gaticule (Fig. 288).

EXAMPLE

Prepare a graticule for Africa on 1 : 50,000,000 scale at 10° interval. Let Africa be bounded by 40°N and 40°S latitude and 20°W and 60°E long-

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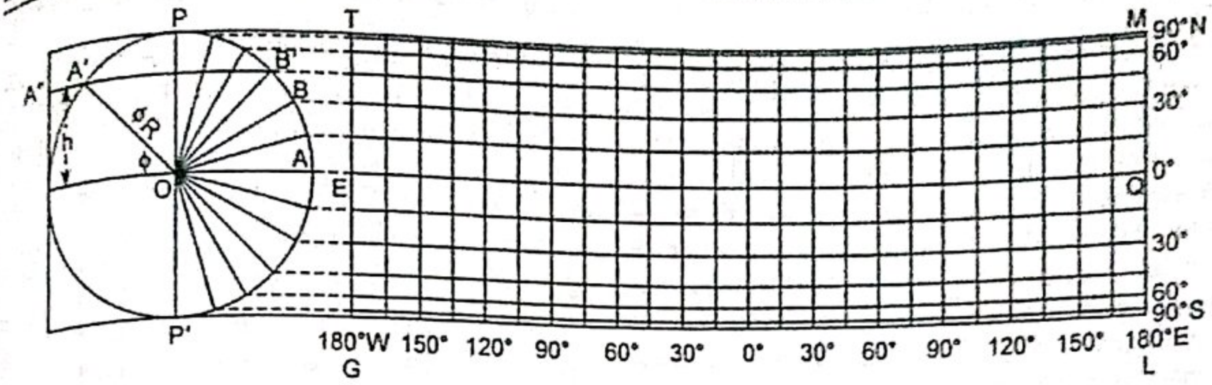

Fig. 288

Trignometrical Construction

According to the given scale the radius of the reduced sphere, *i.e.*,

$$R = \frac{250,000,000}{50,000,000} = 5''.$$

Now with the formula $h = R \sin \phi$, the intervals between the equator and the parallels may be calculated as in the Table 9.

TABLE 9

ϕ	$\sin \phi$	R	h
10	0.17	5"	0.85"
20	0.34		1.70"
30	0.50		2.50"
40	0.64		3.20"

The distance between two meridians

$$= \frac{2\pi R \times d}{360} = \frac{2 \times 22 \times 5 \times 10}{360} = 0.87''$$

Draw EQ and NS from the point O intersecting each other at right angles. Mark-off the points along NS at 0.85", 1.70", 2.50" and 3.20" distance from O. From these points draw the lines parallel and equal to EQ. Starting from O, also mark-off the point along EQ at a distance of 0.87". From these points draw the lines parallel to the central meridian NS. Thus the graticule may be completed (Fig. 289).

For graphical construction the same procedure may be followed as in Fig. 288.

Mercator's Projection

The Mercator's Projection belongs to the cylindrical group of projection. The projection is

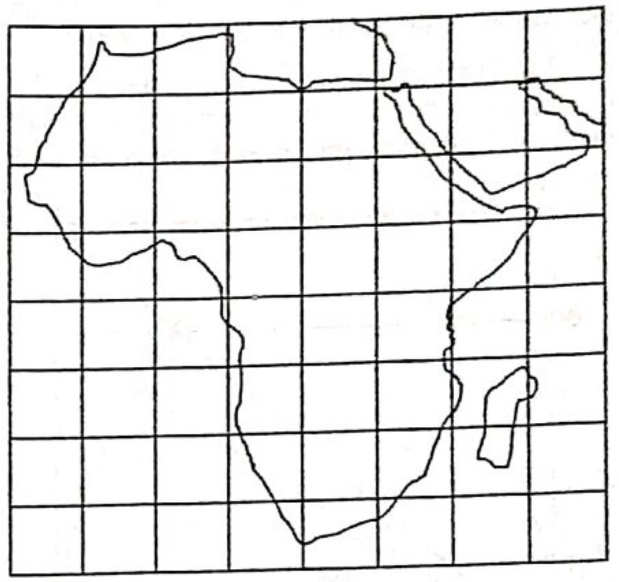

Fig. 289

often called the cylindrical orthomorphic. This is rather one of the most popular projections for the world maps in atlases. Its popularity is great because of its singluar use in navigation. It was first designed by Flemish, a cartographer in 1569 and later modified by Edaward Wright of the Cambridge University. Like other cylindrical projections the meridians and parallels intersect each other at right angles but the distances between the parallels of latitude gradually and proportionately increase towards the area truly. Meridians are equidistant straight lines. The scale is considerably increased towards the poles as all the parallels are of the same length, but it is the same in all directions at any point of intersection of the parallels and meridians because the distances between the parallels are so arranged. As the interval between the parallels increase considerably without distorting the shape, this is why the projection is orthomorphic. On

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account of proportionaley equal exaggeration of parallel and meridian scales at one point not only the shape of small areas in any part of the map is true, but the directions also become correct everywhere. In Fig. 290. *ABCD* is a mesh on the globe and *ABC"D"* is the mesh on projection. It may be noted that the direction of *C* remains the same when projected to *C"* because *CC'* is proportional to *C'C"*. For this small area the shape is magnified, not distorted. But if you take a large area then the shape also may be distorted because the parallel scale changes in different latitudes.

The exaggeration in area is so much so that Greenland on this projection appears to be greater than South America, though in actuality the latter is nine times bigger. While on the parallel of 60° the area is increased four times and on 80°, 33 times. The pole is infinite so that it is futile to show higher latitudes on this projection, and thus the projection is drawn generally upto 80° only. (Fig. 291*a*, *b*).

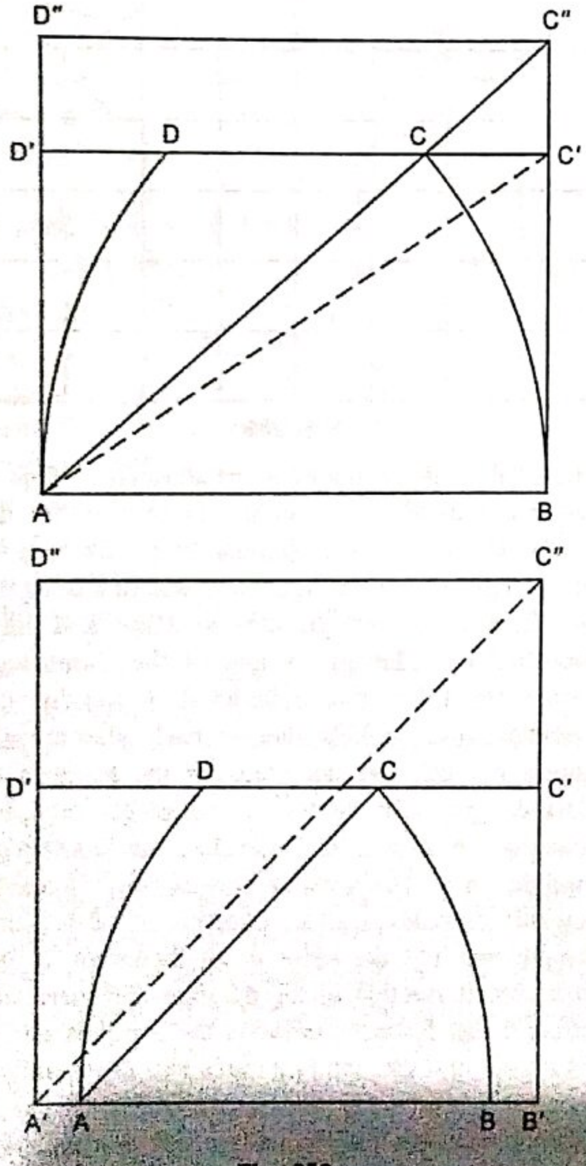

Fig. 290

The value of this projection for sailors is increased by the fact that compass direction may be shown by straight lines. Any straight line drawn on the projection make equal angles with all the parallels and represents a line of constant bearing on the globe and thus forms a "*Loxodrome*" or "*rhumb line*". This due to the fact that all the meridians and parallels intersect at right angles and both the vertical and horizontal scale are balanced. In any other cylindrical projection too, any straight line may make equal angles with parallel and meridians, but it is not a "*rhumb line*" because the parallel and meridian scales are not adjusted. It is, indeed, the adjustment of longitudinal and latitudinal scales that gives the Mercator's Projection this distinction of showing constant bearings. A sailor has simply to plot the bearings on his chart so as to find the route. The bearing from one point to another can be found out by only drawing a line between them and reading off the angle which is made by this line with the meridians. For mathematical determination of sailing and flying courses on the Mercator's Projection Steer's *Study of Man Projection*, (pp. 165-175) may be consulted.

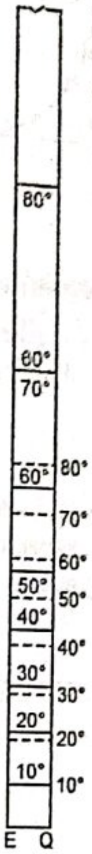

Fig. 291(a)

Here a comparison between the Mercator's and Gnomonic Projections may be drawn. In the former, staight lines correspond with those of constant bearing; on the latter, bearings are not constant but they denote shortest distance between two points and it is due to this fact that Gnomonic charts are tributary to Mercator's charts in navigation along

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great circle routes. But as the amount of distortion increases equally and rapidly outwards in all directions from the centre of the map, it is useful only for a small area, while the Mercator's has won the popularity of nearly the whole world. On Mercator's projection the direction may be marked in the same way all over the map, a quality which makes it suitable for meterological charts also. Thus the Projection is appropriate for the world map showing drainge pattern, routes, ocean, currents, wind system and direction, etc.

Construction

The construction of this projection becomes very simple with the help of the table, but the derivation of the formula is much difficult. If we, however, find out how much a parallel is increased in length, it would be possible to calculate the interval between the parallels and the equator, as we have already noticed that the vertical scale is increased in the same proportional in which the longitudinal or horizontal scale is increased.

In the diagram 290, $\frac{CD}{AB} = \cos\phi$.

It is obvious that a degree of true distance between two meridians along latitude ϕ is equal to a degree of distance between the same along the equator multiplied by $\cos\phi$ or divided by $\sec\phi$. In other words, the length of the lines of latitude on Mercator's increase $\sec\phi$ times its true length. Thus the distance between the parallels and the equator *i.e.*, the vertical scale is also increased $\sec\phi$ times the true scale at every point along the meridional line away from the equator. Now, as this increase will be continuous of every minute and degree away from the equator, the total increase in the distance between any parallel and the equator would be the sum of an infinite series. The distance between any parallel and the equator may, however, be calculated with the help of the following formula and tabulated.

$Y = 2.306\ R \log \tan (45 + \phi/2)$, when Y denotes the distance between any parallel and the equator, ϕ denotes latitudes and R, the radius of the reduced sphere.

EXAMPLE

Draw a graticule for the world map on 1 : 250,000,000 scale at 10° interval.

On the given scale $R = \frac{250{,}000{,}000}{250{,}000{,}000} = 1''$.

The length of the equator

$$= 2\pi R = \frac{2 \times 22 \times 1}{7} = 6.3''$$

The interval between the meridians

$$= \frac{6.3}{36} = 0.175''$$

and $\quad 2.3026\ R = 2.3026''$

Y, the distance between the parallels and the equator is given in Table 10.

TABLE 10

ϕ	$45 + \phi/2$	$(45 + \phi/2)$	Y
10	50	0.07619	0.175″
20	55	0.15477	0.356″
30	60	0.23856	0.549″
40	65	0.33133	0.763″
50	70	0.43893	1.011″
60	75	0.57195	1.317″
70	80	0.75368	1.735″
80	85	1.05805	2.436″

Draw the equator, $EQ = 6.3''$. From the middle point O on it, erect perpendicularly the central meridian NS. Mark off the Y distance from O along ON and OS. From these points draw lines parallel to EQ. Divide EQ at equal distances of 0.175. From these points of division draw the meridians parallel and equal to NS. Thus the construction of graticule may be completed [See Fig. 291(B)].

To avoid trigonometrical calculations the graticule on this projection can be prepared with the help of the table 11 in which ϕ denotes the latitudes; Y, the distance of the parallels from the equator, and R, radius of the reduced sphere.

From the construction of charts and maps on a large scale, more detailed tables showing Y-distances for every minute and degree of parallel, may be found in *Element of Map Projection* by Deetz and Adams.

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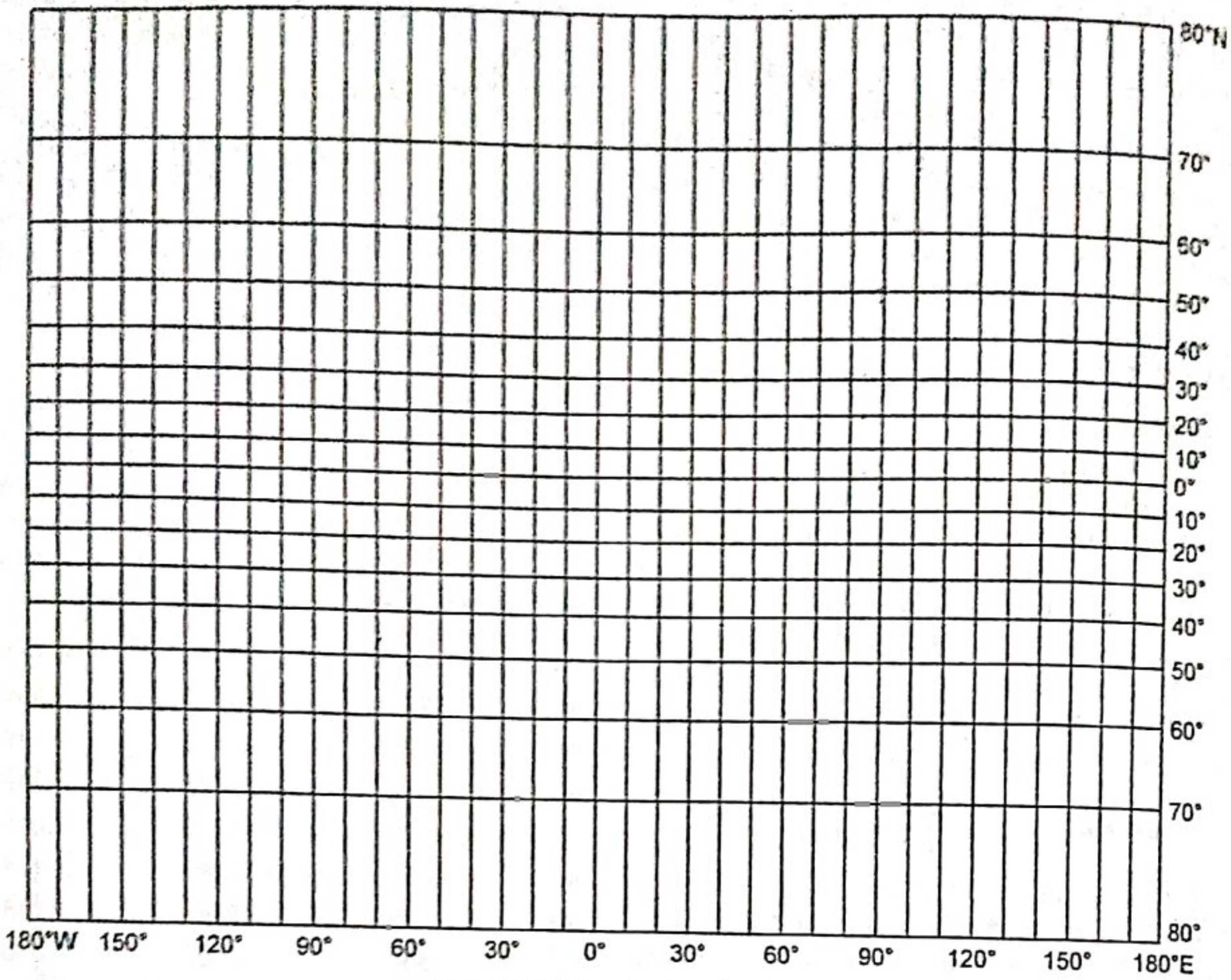

Fig. 291(b) Mercators' Projections

TABLE 11

φ	Y	φ	Y
5°	0.08743 × *R*	50°	1.01069 × *R*
10°	0.17547 × *R*	55°	1.15424 × *R*
15°	0.26475 × *R*	60°	1.31695 × *R*
20°	0.35628 × *R*	65°	1.50424 × *R*
25°	0.45095 × *R*	70°	1.73542 × *R*
30°	0.54929 × *R*	75°	2.02760 × *R*
35°	0.65282 × *R*	80°	2.43624 × *R*
40°	0.76291 × *R*	85°	3.13130 × *R*
45°	0.88136 × *R*	90°	∞

Sinusoidal or Sanson-Flamsteed Projection

This projection has been devised by Sanson, a French cartographer, and Flamsteed, the British Royal Astronomer; hence named after them. It is also called Sinusoidal as the Longitudes present *Sine* curves. It may be regarded as a modification of the cylindrical equidistant projection. The main defect of the latter is that the parallels scale is exaggerated in it due to the fact that all the parallels are equal to the equator in length. While in this projection this defect is eliminated making each parallel true to scale and by dividing it correctly. The Projection may be also treated as a special case on Bonne's when the standard parallel is the equator, and hence all parallels are equidistant straight lines and are drawn true to scale like the equator. The central meridian is a straight line and on a graticule for world map it is only half the length of the equator; other meridians are regular curves drawn by joining the points marked off along the parallels at true spacing. Thus it is also an equal area projection. The globe may be represented on this system but the shape is greatly distorted towards the four corners of the map, because the curvature

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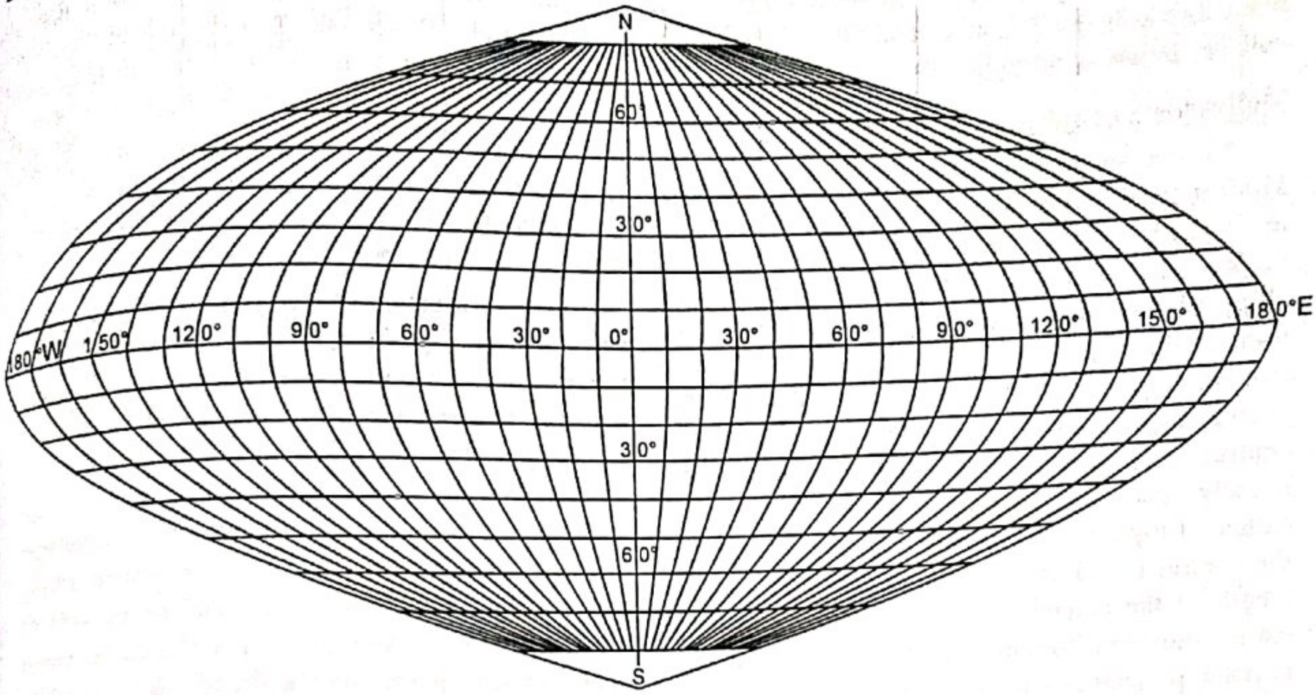

Fig. 292

of the meridians is increased rapidly away from the central meridian (Fig. 292).

The system is, however, best suited for the equatorial countries with small east-west and north-south extent. Thus, in atlases, a map of South America or Africa is generally drawn on this projection.

EXAMPLE

Prepare graticule for South America on 1 : 50,000,000 scale, at an interval of 10°.

$$R = \frac{250,000,000}{50,000,000} = 5''.$$

Length of Equator

$$= 2\pi R = \frac{2\times22\times5}{7} = 31.4''$$

The length of the central meridian

$$= \frac{31.4}{2} = 15.7''$$

The true distance at 10° interval

$$= \frac{31.4\times10}{360} = 0.87''$$

Mark off these points along the equator as well as along the central meridian at a distance of 0.87". From the point thus marked along the central meridian, draw the lines of latitude parallel to the equator. Divide the parallels equally at 10° interval when their length is $2\pi R \cos\phi$ just as in the Bonne's and with the help of the Table 12 complete the construction likewise.

TABLE 12

R	ϕ	$\cos\phi$	Y	$X = (Y\cos\phi)$
5"	10	0.98	0.87	0.85"
5"	20	0.94	0.87	0.82"
5"	30	0.87	0.87	0.76"
5"	40	0.77	0.87	0.67"
5"	50	0.64	0.87	0.56"
5"	60	0.50	0.87	0.44"
5"	70	0.34	0.87	0.30"
5"	80	0.17	0.87	0.15"

In the Table 12, Y denotes the distance between the parallels and X, the given interval between the meridians along the respective parallels. The table gives data to construct the graticule for the world map. For the map of South America only a few lines of latitude and longitude will be drawn (Fig. 293).

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It is, therefore, obvious that all meridians form ellipses [See Fig. 296(*a*)]. To avoid trignometrical calculations, a beginner may follow the following table* in which *X* denotes the distance between the parallels and the equator ϕ be latitude, θ= the angle in the cenral circle of the ellipse corresponding to ϕ and *R* = radius of the reduced sphere (Table 15).

In Mollweides projection let ϕ be any latitude and ϕ the angle in the central circle of the ellipse corresponding to ϕ.

The parallel can be drawn with the help of either column two or three. Say, the ϕ of 40° is to be drawn. Draw a line to an angle of 32°04′, which meets the central circle at *L*; from *L* draw a line parallel to the equator, which will be the required parallel. Similarly the parallal can be drawn at a distance of *R* × 0.7508 from the centre along the central meridian.

Graphical Construction

At first from the calculated radius a circle, *NESQ* [Fig. 296(*b*)] and two axes of *NE* and *EQ* to be drawn. The radius of *NE* = *R*√2 taken to drawn the longitudes both the sides representing 90° longitudes

TABLE 15. Distances of the parallels from the equtor

I ϕ	*II* θ	*III* *Distance from equator*
5°	3°56′	0.09701 × *R*
10°	7°52′	0.19356 × *R*
15°	11°49′	0.28961 × *R*
20°	15°47′	0.38466 × *R*
25°	19°47′	0.47866 × *R*
30°	23°50′	0.57145 × *R*
35°	27°55′	0.66211 × *R*
40°	32°04′	0.75080 × *R*
45°	36°18′	0.83722 × *R*
50°	40°38′	0.92094 × *R*
55°	45°05′	1.00144 × *R*
60°	49°41′	1.07830 × *R*
65°	54°28′	1.15085 × *R*
70°	59°32′	1.21894 × *R*
75°	64°58′	1.28136 × *R*
80°	70°59′	1.33704 × *R*
85°	78°03′	1.38354 × *R*
90°	90°0′	1.41420 × *R*

*The table has been reproduced from Elements of *Map Projection* by Steers, J.A., p. 152.

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on which equatorial and polar axes to be fixed. From the equatorial axes the axis to be extened both the sides equally. Further, with the help of table 15 the longitudinal distances on a particular latitude to be calculated and parallel to equator have to be drawn. Similar to that of equator outer circles are drawn on the basis of doubling the equatorial distance, representing latitudes. Afterward at the given interval of 20° in total 18 longitudes to be drawn with the help of respective curves. Thus, finally the graticule will be ready.

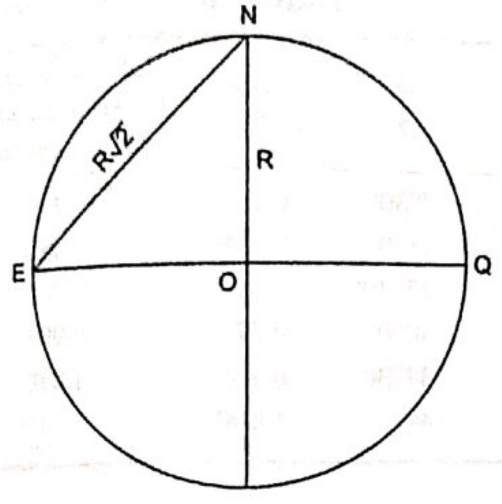

Fig. 296 (*b*)

Gall's Projection

This is a stereographic cylindrical projection quite akin to Mercator's but it differs from the latter in that it is not orthomorphic. In Gall's the distance between the parallels is reduced to avoid too much exaggeration of area towards the poles; while in Mercator's the distance between the parallels increases proportionately so that shape may be truly preserved. Gall's projection is also not equal in area, but as the distortion in higher latitudes is not much, it is used for general world maps in preference to other cylindrical projections.

This projection is made on a cylinder which is supposed to pass through the globe halfway between the equator and the poles; that is, the cylinder cuts through the sphere along the 45°N and 45°S parallels, parallel to the polar axis. The parallels are then, projected stereographically (Fig. 297). The meridians, as in all cylindrical projections, are equidistant vertical straight lines. The 45° N and 45°S parallels are true to sacle and all other parallels are equal to it in length. Thus from these two parallels meridian and parallel scales decreases towards the equator and increase towards the poles.

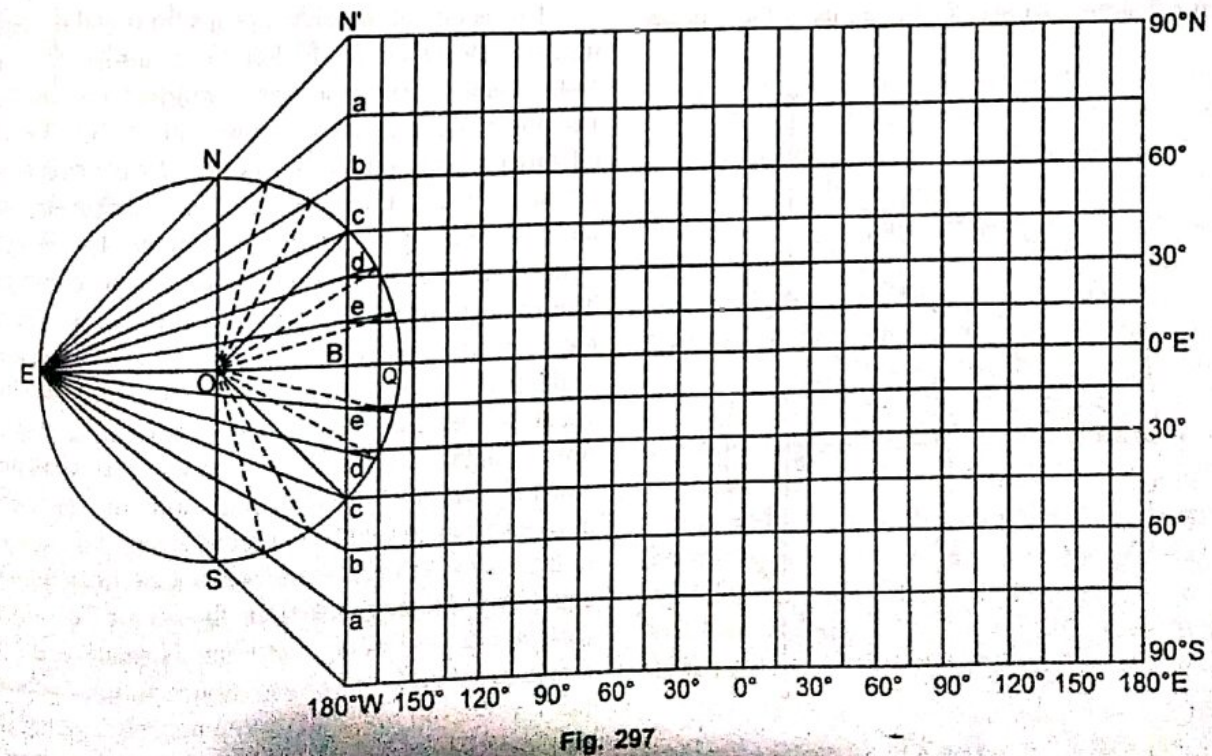

Fig. 297

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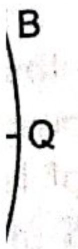
points, two marked on the circumference and one on the central meridian. It may be noted that the lines *EQ* and *PS*, (produced if necessary), will form the loci of the centres with which the arcs of circles, representing meridians and parallels respectively are to be drawn (See Fig. 300). The centres may be graphically located as stated.



International Map Projection

This is a modified polyconic projection. Following the decision of International Map Committee held in 1909, the projection was introduced for the topographical maps of the whole world on a scale of 1 : 1,000,000 in preference to polyconic for the following reasons :

(1) In the polyconic, sheets east and west of each other do not fit together properly because the marginal meridians are curved; while in the International Projection thay have been made to fit on all sides (See Fig. 301). In the former, as we have seen in the foregoing, all the parallels are equally and correctly divided into equal parts and the meridians are drawn as regular curves by joining the points of division thus obtained. Whereas in the latter only the top and bottom parallels are divided into equal parts at true distances and the meridians are drawn as straight lines, passing through the corresponding points thus marked along them. As the marginal meridians become straight lines, the adjoining sheet on all sides may be fitted together with little distortion. But in doing so the intermediate parallels become very slightly short.

(2) In the polyconic, the scale is correct along the central meridian as it is divided truly, and along

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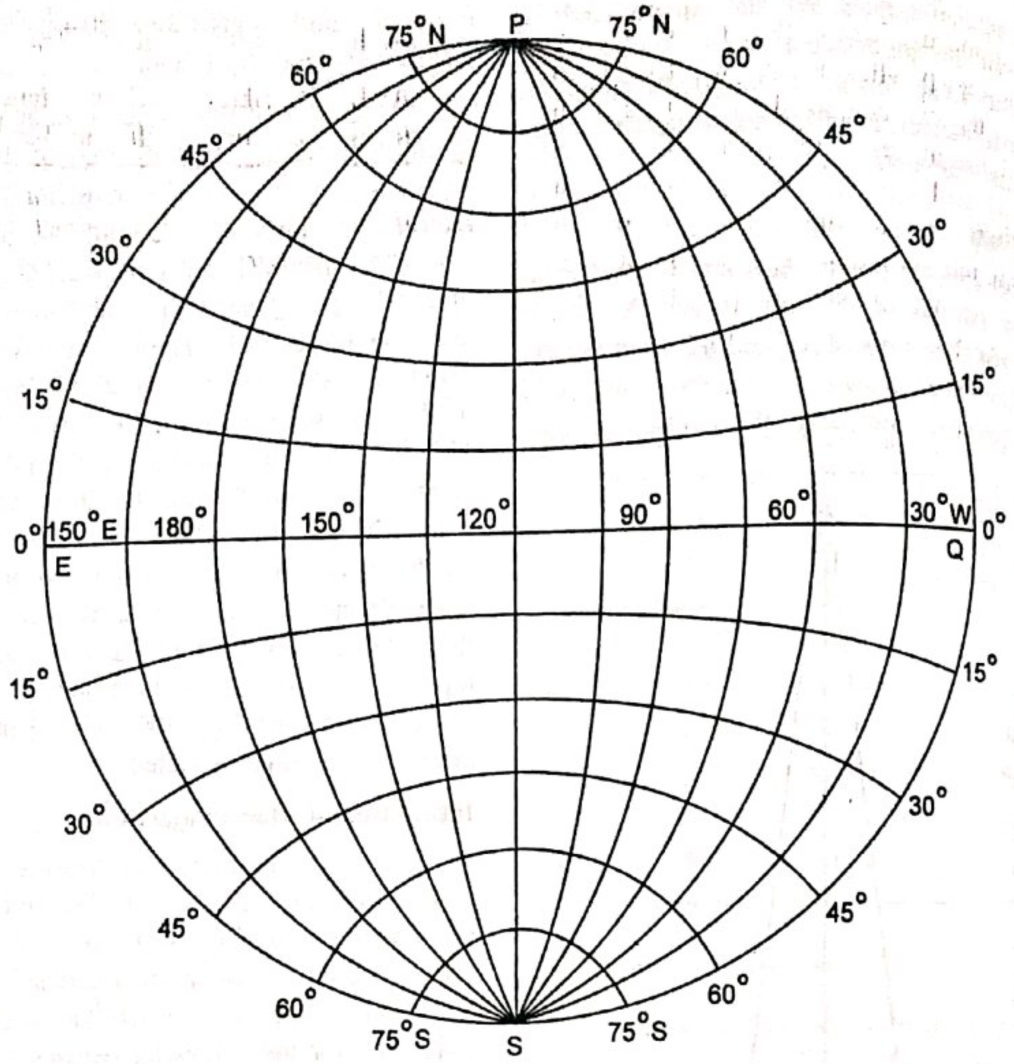

Fig. 300

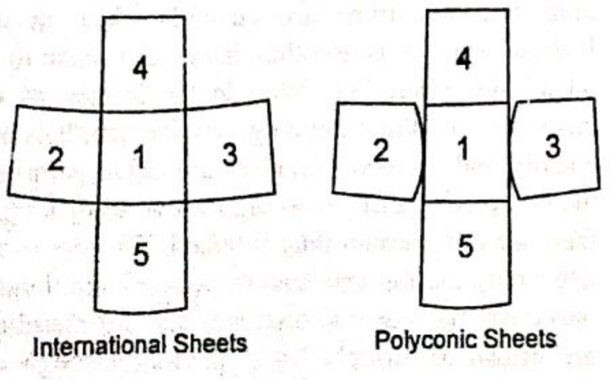

Fig. 301

other meridians it becomes gradually enlarged towards the margins. In the International Map Projections this defect has been minimised by making the scale true along two meridians on each sheet, one lying 2° west and the other 2° east of the central meridian. Thus the scale along the meridian becomes slightly too small and that along the marginal meridians, slightly too long. The parallels are true distances apaert only along the two true meridians.

Each sheet is constructed independent of the other adjacent sheets, with its own central and bounding meridians and parallels. It extends over 4° of latitude and 6° of longitude between 60° N and S of the equator; while between 60° and 88°N and S it covers 4° of latitude and 12° of longitude. The polar maps are circular 4° in diameter. It may be noted that every sheet, for all practical purposes, reserves sufficiently both the quality of orthomorphism and equal area. Moreover, the construction of the graticule on this projection is simple. As in the ordinary polyconic, the parallels are not

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which in doing ... very

per cent.

3. Block Piling Method

This method involves the piling up of a number of unit cubical one above the other in such a way that each one of them may be easily counted. This also produces three-dimensional effect. Its one side is deeply shaded or pitched so that the block may become impressive. It is a simplified form of three dimensional blocks, provided by cubes, spheres cylinders, etc. The difficulty of finding out the volumes of various geometrical forms required to make them proportionate to the quantity represented, has been avoided in this method by assuming a

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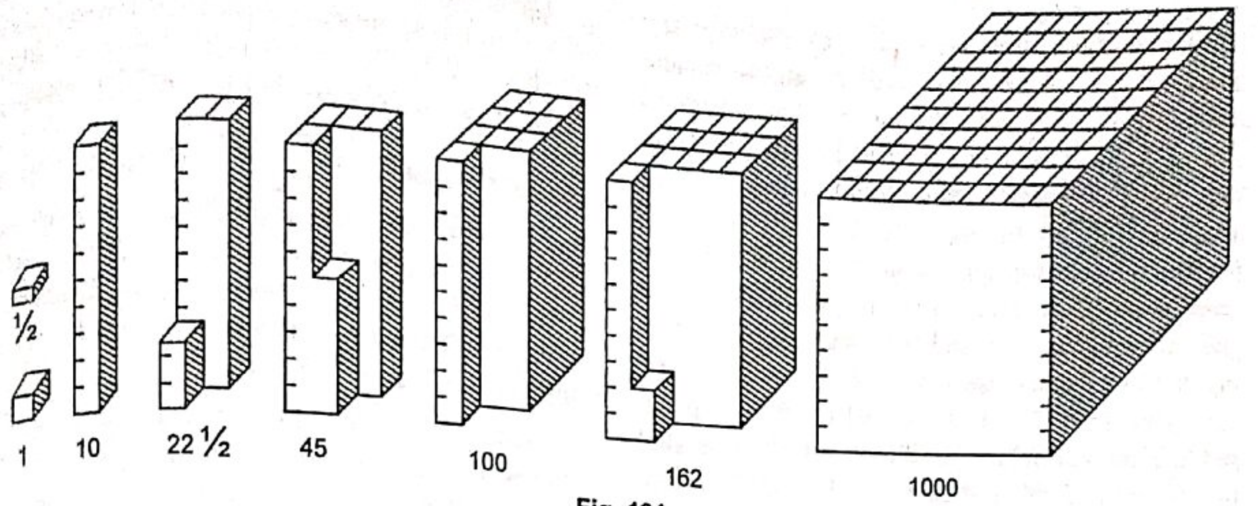

Fig. 191

small unit cube to represent a certain quantity. The such unit cubes may be piled one above the other to produce one full block, one side of which may be sub-divided into 10 equal parts, and thus, one sub-division may represent one unit. In Fig. 191, such piled blocks showing 1/2 unit, 1 unit, 10 units, 22.5 units, 45 units, 100 units, 162 units and 1000 units may be noted.

This method is advantageous in comparison with others in the following ways : (*i*) It is more impressive; (*ii*) the full meaning of the block can be understood easily because the sub-divisions are marked on the block itself, which facilitate exact measurement of the quantity represented by it; (*iii*) it occupies less space than a bar, or rectangular block of circle, a quality which makes it suitable for being used as symbols in cartograms and distribution maps (See Fig. 225). But it may be noted here that a sphere requires minimum of space for the maximum of volume. So in maps where we cannot afford to give more space for a small volume, spheres may be preferred to these blocks; but it is easier to construct the latter than the former and moreover, calculation of radii of spheres presents further difficulty; this restricts the use of spheres to very specific end.

EXAMPLE

Diagrammatically represent the data given in Table 5 by block-pile method :

TABLE 5. The production of some crops in India in 1970-71

Crops	*Production in Metric Tons*
Cotton	4555.7
Jute	4905.2
Tobacco	350.0

Let one-tenth inch cube represent 10 metric tons. Then the number of unit cubes will be 455.6 for cotton, 490.5 for jute and 35.0 for tobacco. The

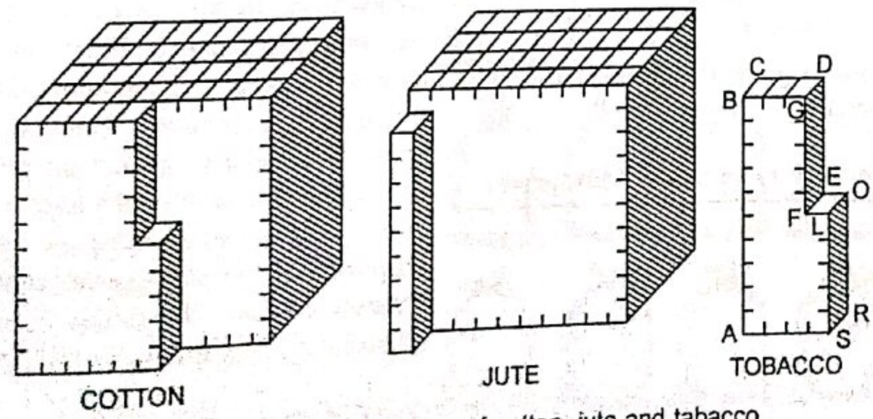

Fig. 192. Showing production of cotton, jute and tabacco.

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whole numbers may be easily represented; the fractions may either be deleted or approximately represented. Let us take the tobacco block first. In this case 30 cubical blocks may be piled together one above the other and 5 unit cubes may be attached with it. In Fig. 192 *AB* is of one inch length. Ten sub-divisions at an interval of 1/10″ are marked on it. *BG* and *AS* are perpendicular to *AB* and are equal to 0.3″ and 0.4″ respectively, *GF* is parallel to *AB* and equal to 0.5″. *FL* is parallel to *AB* and equal to 0.1″. *CD* is parallel to *BG* and equal to 0.3″, *DE* and *OR* are parallel to *AB*. *SR* is parallel to *LO*, *FE* and *BC* and each equal to 1/10″ except *BC*. Sub-divide *AB*, *BG*, *CD*, GF and *AS* at 1/10″ interval. Thus, the block *ABCDEORS* may be obtained to represent 350 tons of tobacco. The block gives a visual picture of the pile of tobacco and its quantity is measurable to the decimal point. Similarly, blocks of other production figures may be made (Fig. 192).

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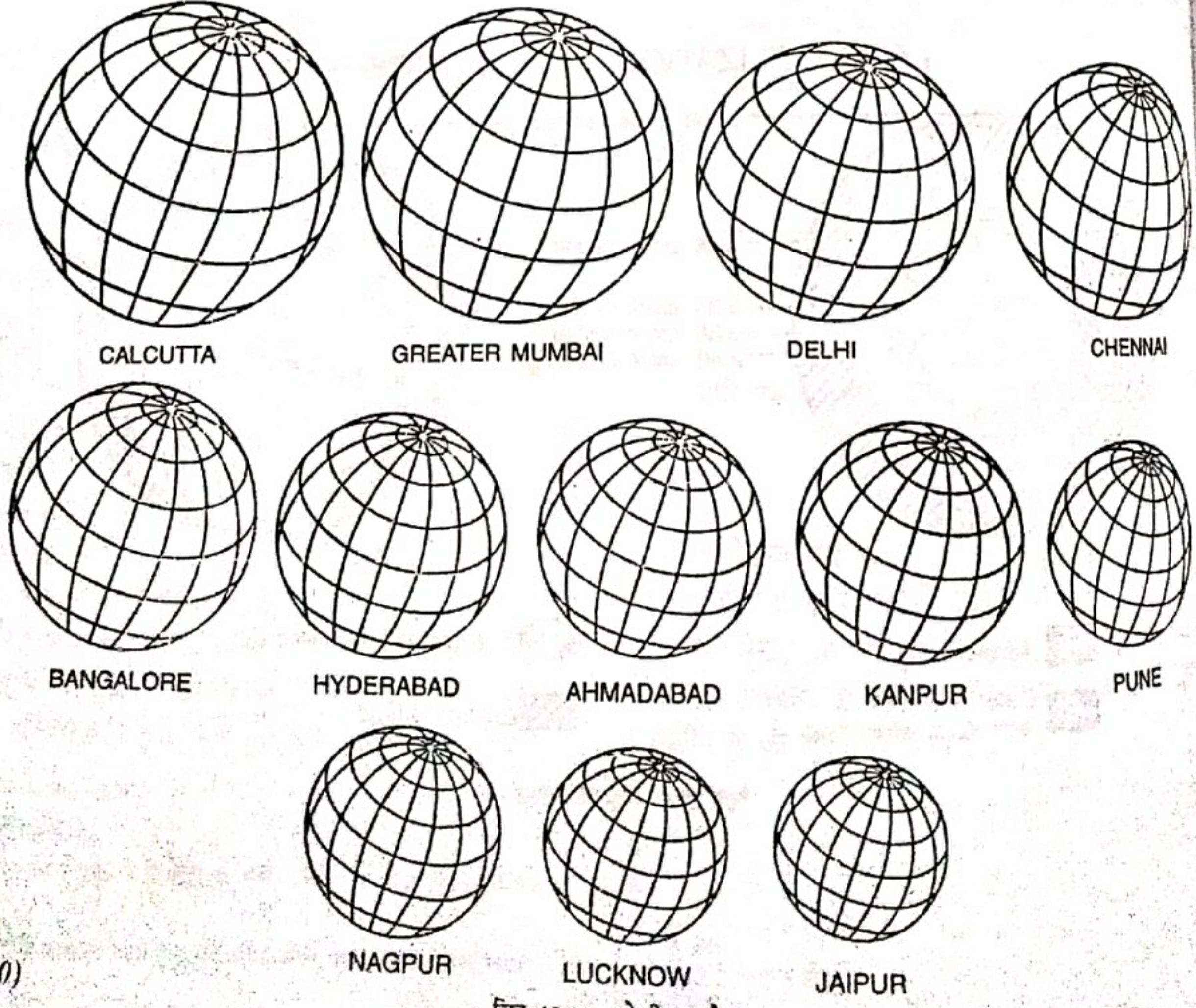
त्रिविम आरेख
(Three-Dimensional Diagram)

त्रिविम आरेख में लम्बाई, चौड़ाई व ऊँचाई तीनों विस्तारों का प्रयोग होता है अर्थात् दिये हुए मूल्यों को घनों (cubes), ब्लॉकों (blocks), अथवा गोलों (spheres) आदि के आयतन (volume) के अनुपात में प्रदर्शित किया जाता है। अतः त्रिविम आरेखों को **आयतन-आरेख** (volume diagram) भी कहते हैं। इस प्रकार के आरेख प्रायः उस दशा में बनाये जाते हैं जब दिये हुए मूल्यों में अन्तर बहुत अधिक होता है अर्थात् एक पदमूल्य का मान बहुत अधिक तथा दूसरे का बहुत कम होता है। प्रमुख प्रकार के त्रिविम आरेखों को नीचे समझाया गया है।

[I] गोलीय आरेख
(Spherical diagram)

गोलीय आरेख वृत्तारेखों से दो बातों में भिन्न होते हैं—**प्रथम**, गोलीय आरेख में वृत्तों के बजाय गोले खींचे जाते हैं तथा **द्वितीय**, इन गोलों के अर्द्धव्यास दिये हुए मूल्यों के [illegible] (cube roots) के अनुपात में होते हैं। किसी संख्या [illegible] घनमूल ज्ञात करने के लिये उस संख्या के [illegible] (logarithms) में 3 का भाग देते हैं। भाग देने से [illegible] भजनफल का प्रतिलघुगणक (antilogarithms) उस संख्या [illegible] घनमूल को प्रकट करेगा। लघुगणक व प्रतिलघुगणक ज्ञात [illegible] की विधि को अध्याय 24 में विस्तारपूर्वक उदाहरण [illegible] समझाया गया है।[4] घनमूल ज्ञात हो जाने पर वलय आरेख [illegible] बतलायी गई विधि की तरह भिन्न-भिन्न संख्याओं को [illegible] करने वाले गोलों के अर्द्धव्यासों की गणना कर लेते हैं [illegible] पहले न्यूनतम या अधिकतम संख्या के घनमूल को प्रकट [illegible] वाले गोले का सुविधानुसार कोई अर्द्धव्यास मान लेते हैं। [illegible] बाद अन्य गोलों के इसी अनुपात में अर्द्धव्यास ज्ञात कर लि[illegible] जाते हैं। जनसंख्या का वितरण प्रदर्शित करने वाले बिन्दु[illegible] मानचित्रों (dot maps) में बड़े-बड़े नगरों की जनसंख्या [illegible] करने के लिये प्रायः इस प्रकार के गोले खींचे जाते हैं।

POPULATION OF
METROPOLITAN CITIES OF INDIA, 1981



चित्र 13.18—गोलीय आरेख।

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उदाहरण (18) निम्नलिखित आँकड़ों को गोलीय आरेख के द्वारा प्रकट कीजिये :

भारत के महानगरीय शहरों की जनसंख्या, 1981

महानगर	जनसंख्या	महानगर	जनसंख्या	महानगर	जनसंख्या
कलकत्ता	9,165,650	बंगलौर	2,913,537	पुणे	1,685,266
बृहत् मुम्बई	8,202,759	हैदराबाद	2,565,536	नागपुर	1,297,977
दिल्ली	5,227,730	अहमदाबाद	2,515,195	लखनऊ	1,006,843
चेन्नई	4,276,635	कानपुर	1,685,308	जयपुर	1,004,669

दिये हुए आँकड़ों को निकटन संख्याओं में लिखकर निम्न प्रकार गोलों के अर्द्धव्यास ज्ञात कीजिये :

महानगर	जनसंख्या	निकटन संख्या (लाख)	घनमूल	गोले का अर्द्धव्यास (सेमी)
कलकत्ता	9,165,650	91.66	4.505	= 2.0
बृहत् मुम्बई	8,202,759	82.03	4.345	$\frac{4.345}{4.505} \times 2 = 1.93$
दिल्ली	5,227,730	52.28	3.740	$\frac{3.740}{4.505} \times 2 = 1.66$
चेन्नई	4,276,635	42.77	3.497	$\frac{3.497}{4.505} \times 2 = 1.55$
बंगलौर	2,913,537	29.14	3.077	$\frac{3.077}{4.505} \times 2 = 1.37$
हैदराबाद	2,565,536	25.66	2.949	$\frac{2.949}{4.505} \times 2 = 1.31$
अहमदाबाद	2,515,195	25.15	2.932	$\frac{2.932}{4.505} \times 2 = 1.30$
कानपुर	1,685,308	16.85	2.564	$\frac{2.564}{4.505} \times 2 = 1.14$
पुणे	1,685,266	16.85	2.564	$\frac{2.564}{4.505} \times 2 = 1.14$
नागपुर	1,297,977	12.98	2.351	$\frac{2.351}{4.505} \times 2 = 1.04$
लखनऊ	1,006,843	10.07	2.159	$\frac{2.159}{4.505} \times 2 = 0.96$
जयपुर	1,004,669	10.05	2.158	$\frac{2.158}{4.505} \times 2 = 0.95$

उपरोक्त सारणी में अधिकतम जनसंख्या वाले महानगर (कलकत्ता) की जनसंख्या के घनमूल को प्रकट करने वाले गोले का सुविधानुसार 2 सेमी अर्द्धव्यास मानकर, अन्य महानगरों की जनसंख्याओं को प्रदर्शित करने वाले गोलों के अर्द्धव्यासों की गणना की गई है। अब इन अर्द्धव्यासों के अनुसार गोले खींचकर आरेख पर शीर्षक आदि लिखिये (चित्र 13.18)।

4. किसी संख्या के घनमूल को लघुगणक व प्रतिलघुगणक सारणियों की सहायता से ज्ञात करते हैं। परन्तु यदि केवल गोलीय आरेख बनाने के उद्देश्य से दी हुई संख्याओं के घनमूल ज्ञात करने हों तो पुस्तक के अन्त में दिये गये परिशिष्ट–5 की सहायता ली जा सकती है। इस परिशिष्ट में 1 से 1000 तक की संख्याओं के घनमूल लिखे गये हैं। इस परिशिष्ट को प्रयोग करने के लिये पहले दी गई संख्याओं को 3 अंकों वाली संख्याओं में परिवर्तित कर लेते हैं। उदाहरणार्थ, मान लीजिये 53,635,000 72,340,000 तथा 35,833,000 कोई तीन संख्याएँ है, जिनके घनमूल ज्ञात करने हैं। इन संख्याओं को तीन अंकों वाली संख्याओं में क्रमशः 536 लाख 723 लाख तथा 358 लाख लिखा जा सकता है। अब परिशिष्ट में 536 723 व 358 के घनमूल पढ़िये, जो क्रमशः 8.1231, 8.9752 व 7.1006 के बराबर हैं।

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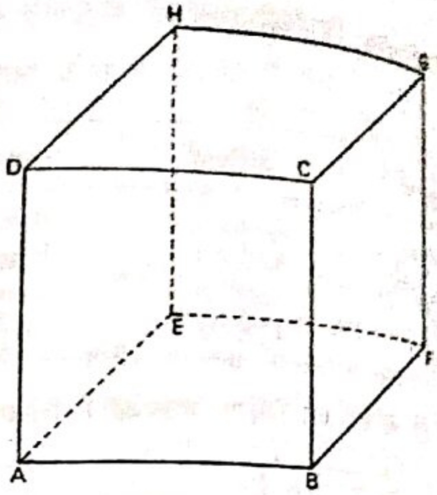
[II] घनारेख

(Cube diagram)

इन आरेखों में दिये हुए मूल्यों को घनों (cubes) के द्वारा प्रदर्शित किया जाता है। इन घनों के आयतनों (volumes) में वही अनुपात रखा जाता है जो अनुपात उन मूल्यों के घनमूलों में होता है। अतः आरेख बनाने के लिये सर्वप्रथम दिये हुए मूल्यों के घनमूल ज्ञात किये जाते हैं। तत्पश्चात् न्यूनतम अथवा अधिकतम मान वाले घनमूल को प्रकट करने वाले घन की भुजा को सुविधानुसार निश्चित करके उपरोक्त उदाहरण में बतलायी गयी विधि के अनुसार, शेष घनों में प्रत्येक की एक भुजा ज्ञात कर लेते हैं। चूँकि घन में लम्बाई, चौड़ाई व ऊँचाई तीनों माप एक समान होती हैं अतः किसी घन की एक भुजा ज्ञात हो जाने पर उस घन को सरलतापूर्वक बनाया जा सकता है। चित्र 13.19 A में घन बनाने की विधि को समझाया गया है। मान लीजिये, AB कोई भुजा है जिस पर घन बनाना है। AB भुजा पर ABCD वर्ग खींचिये। इस वर्ग के केन्द्र E से दायीं ओर को अथवा बायीं ओर को (चित्र में दायीं ओर को) AB के समान्तर व बराबर EF रेखा खींचकर EFGH दूसरा वर्ग बनाइये। अब B, C व D बिन्दुओं को क्रमशः F, G व H बिन्दुओं से मिलाते हुए BF, CG व DH सरल रेखाएँ खींचिये। चित्र में



चित्र 13.19 A

बिन्दुदार रेखाएँ बनाने का उद्देश्य केवल घन की आकृति को स्पष्ट करना है तथा आरेख में इन रेखाओं का प्रदर्शन अनावश्यक है।

उदाहरण (19) निम्नलिखित आँकड़ों को घनारेख के द्वारा प्रदर्शित कीजिये।

भारत में प्रमुख खनिजों के अनुमानित संचित भण्डार (करोड़ टन)

खनिज	संचित भण्डार	खनिज	संचित भण्डार
कोयला (coal)	8,577	ताम्र-अयस्क (copper ore)	40
लोह-अयस्क (iron ore)	2,300	सीसा-जस्ता अयस्क (lead zink ore)	21
बॉक्साइट (bauxite)	240	मैंगनीज़ अयस्क (manganese ore)	8

घनारेख बनाने के लिये निम्न प्रकार गणना कीजिये :

खनिज	संचित भण्डार (करोड़ टन)	घनमूल	घन की भुजा (सेमी)
कोयला	8,577	20.46	$\frac{20.46}{2} \times .25 = 2.56$
लोह-अयस्क	2,300	13.20	$\frac{13.20}{2} \times .25 = 1.65$
बॉक्साइट	240	6.21	$\frac{6.21}{2} \times .25 = 0.77$
ताम्र-अयस्क	40	3.42	$\frac{3.42}{2} \times .25 = 0.43$
सीसा-जस्ता अयस्क	21	2.76	$\frac{2.76}{2} \times .25 = 0.35$
मैंगनीज़ अयस्क	08	2.00	=0.25

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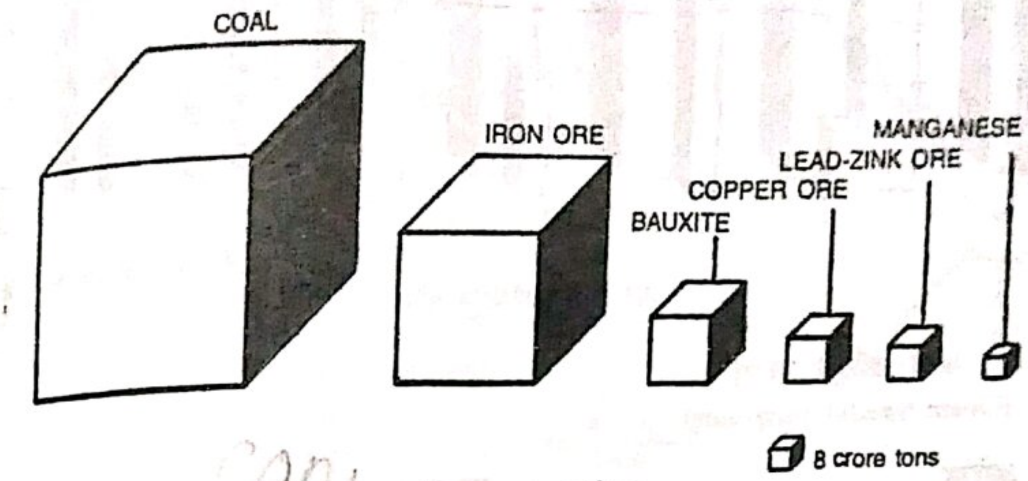
उपरोक्त सारणी में न्यूनतम घनमूल (2.0) प्रकट करने वाले घन की भुजा को सुविधानुसार 0.25 सेमी मानकर अन्य घनों की भुजाएं ज्ञात की गई हैं। अब पहले बतलायी गई विधि के अनुसार 2.56, 1.65, 0.77, 0.43, 0.35 व 0.25 सेमी भुजा वाले घन बनाकर क्रमशः कोयला, लौह-अयस्क, बॉक्साइट, ताम्र-अयस्क, सीसा-जस्ता अयस्क व मैंगनीज़ अयस्क के संचित भण्डारों को प्रकट कीजिये (चित्र 13.19 B)।

MINERAL RESERVES OF INDIA



चित्र 13.19 B—घनारेख।

(iii) ब्लॉक-पुंज आरेख

(Block pile diagram)

इस आरेख में किसी मूल्य को प्रकट करने के लिये पूर्व निश्चित मापनी के अनुसार समान आकार वाले घनों की संख्या ज्ञात करके उन्हें एक पुंज के रूप में बनाते हैं। उदाहरणार्थ, यदि 1 घन=5 टन की मापनी मानी गई है तो 100 टन प्रदर्शित करने के लिये आरेख में उसी आकार के 100÷5=20 घनों को मिलाकर पुंज बनाया जायेगा। प्रत्येक पुंज में घनों को इस प्रकार सजाते हैं कि उनकी सरलतापूर्वक गिनती की जा सके। अतः पुंज में घनों को 10-10 के कॉलमों में रखना सुविधाजनक रहता है। मान लीजिये, किसी पुंज में 400 छोटे घन दिखलाने हैं तो स्पष्ट है कि पुंज में 10-10 घनों के कुल 40 कॉलम होंगे। अब इन कॉलमों को सुविधानुसार 10×4 अथवा 8×5 की कतारों में रखकर पुंज बनाया जा सकता है। यदि 10-10 घनों के कॉलम बनाने पर कुछ घन शेष बच जाते हैं तो इन बचे हुए घनों के अपेक्षाकृत छोटे कॉलम को पुंज के सामने वाले पार्श्व पर बनाना चाहिए जिससे उसके प्रत्येक घन को गिना जा सके।

उदाहरण (20) निम्नलिखित आँकड़ों की सहायता से एक ब्लॉक-पुंज आरेख बनाइये :

संसार में मत्स्य उत्पादन, 1978

देश	उत्पादन (लाख मीटरी टन)	देश	उत्पादन (लाख मीटरी टन)
जापान	110	नार्वे	35
सोवियत संघ	102	संयुक्त राज्य अमेरिका	31
चीन	70	भारत	25
पीरू	44	डेनमार्क	21

आरेख बनाने के लिये कोई उचित मापनी (मान लीजिये 1 इकाई घन=1 लाख मीटरी टन) निश्चित करके प्रत्येक देश के मत्स्य उत्पादन को प्रकट करने वाले घनों की संख्या ज्ञात कीजिये। उपरोक्त मापनी के अनुसार जापान, सोवियत संघ, चीन, पीरू, नार्वे, संयुक्त राज्य अमेरिका, भारत व डेनमार्क के मत्स्य उत्पादनों को क्रमशः 110, 102, 70, 44, 35, 31, 25 व 21 घनों के द्वारा प्रकट किया जायेगा। अब प्रत्येक देश के घनों को 10-10 के कॉलमों में रखकर समान दूरी के अन्तर पर आठ पुंज बनाइये तथा प्रत्येक पुंज पर सम्बन्धित देश का नाम लिखिये (चित्र 13.20)।

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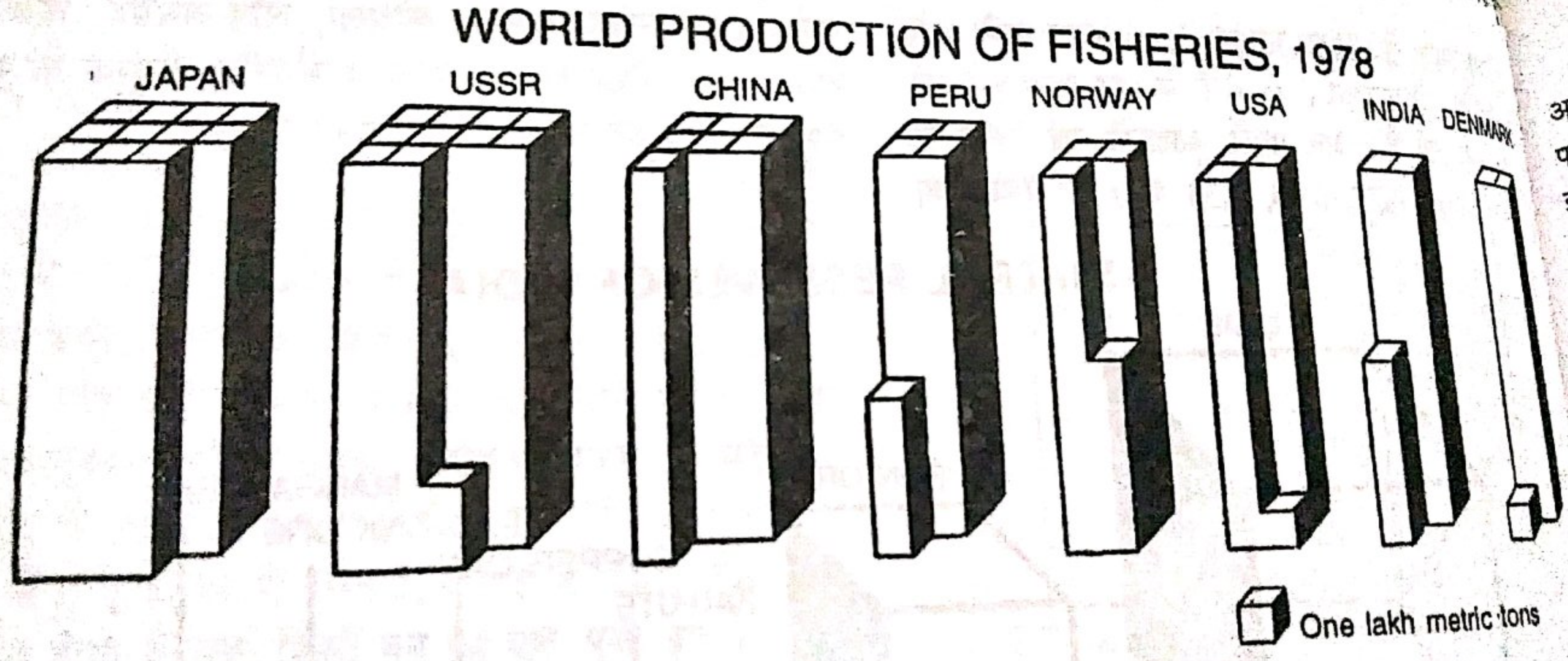

चित्र 13.20 — ब्लॉक-पुंज आरेख।

अन्य विशिष्ट आरेख
(Other Special Diagrams)

...र्ष में पवन कितने-कितने दिन किस-किस दिशा...

Measure a length BA and at A and B erect perpendiculars BD and AE respectively. On these perpendiculars locate by trial two points F and G so that points C, F and G are in one line.

The traiangles AGC and BFC are similar. So that

$$BC = \frac{BF \times AB}{AG - BF}.$$

Plane-Table Surveying

For plane-table survey the following equipments are generally necessary :

(1) The Plane-table with a tripod stand.

(2) An alidade or sighting-rule.

(3) A chain or tape for measuement.

(4) A spirit level.

(5) A trough-compass, also known as a box compass.

(6) A few ranging rods and wooden pegs.

(7) A pair of field glasses for identifying distant objects.

(8) A plumb-bob.

(9) Drawing essentials—paper, pencil, rubber, drawing pins, a pen-knife, pencil sharpener, plotting scale.

Fig. 319

(1) The Plant-Table

It is a light flat drawing board supported on a tripod and this board can be rotated and fixed in any desired horizontal position. (See Fig. 319). The table-top is commonly made up of two pieces of well-seasoned pinewood and the size varies form, say, 15″ × 10″ to 30″ × 24″. The board is supported below by battens with slot holes providing space for expansion and construction. In the centre below the board there is a brass plate with a bossed head which fits into a hole in the centre of the head of the tripod stand and can be kept tight by a wing-nut. The head of the tripod stand is essentially a three-winged thick piece of wood with three legs attached to the wings.

(2) Alidade

It is a strong flat ruler with perfectly straight and parallel edges. At each end there are flap sights which can be folded down when not in use. One of the flaps has a slit in the centre and the other has a vertical thread. Sights are taken along the thread-line. The line joining the thread-line and the centre of the silt is either perfectly midway between the edges or directly over either of the two edges. The edges of the alidade may be graduated as in a scale. Telescopic alidades are also used for surveying large

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area. Here vertical angles can be read and sight vanes are removed from the scale.

(3) Chain or tape

This is used for measuring distances and special care should be taken in measurement when only one base line is used. Tape is helpful in frequent offset measurements which would give too oblique intersection and the tape is also of use in taking check measurements.

(4) Spirit level

This is essential for levelling the table-top. Sometimes the spirit level is laid into the body of the alidade.

(5) Trough compass

It is essentially an oblong box with parallel sides and glass lid cover, carrying a magnetic needle pivoted in the centre. The ends of the needle mover freely over graduated arcs. The magnetic needle can be fixed tight by a screw attached to the pivot or it can be loosened to move freely when taking observation. When the freely moving ends of the needle come to rest with both ends pointing at zero, the axis of the needle is parallel to the sides of the box and thus lines drawn along the edges would shown the magnetic direction.

(6) Ranging rods and pegs

Wooden pegs are necessary for making the stations and ranging rous are used to facilitate taking sights. For example, where one has to mark field boundries, the edges could be marked by ranging rods.

(7) Field glasses

They are of use in getting a clear view of distant object to facilitate in the selection of sights.

(8) Plumb-bob

This is used to centre the plane-table over the station.

(9) Drawing equipment

In accurate plane-table work the paper must be selected with care. It should be a drawing paper with a good surface and be least affected with changes in the humidity of the atmosphere. In very damp weather, celluloid sheets may be used. Tinted papers are less staining to the eyes. The paper should be larger on all four sides than the board. The drawing pencils should not be soft and at least one should have a long thin chisel point for drawing sight lines.

In addition to the above equipments, a water-proof cover also may be required.

Procedure for a complete survey

Before proceding to set up the table of survey it is always desirable to check that all parts of the table are in good working order. The drawing paper should be mounted carefully so as to give a smooth surface. As stated before, the paper should be larger than the board so that the paper can be folded over and pinned or pasted below the board. To get a good result it is advisable to dip the paper in clean water and lay it flat on the board with its drawing surface upwards. The edges could be folded over and pasted below, and the board allowed to dry in a cool pace.

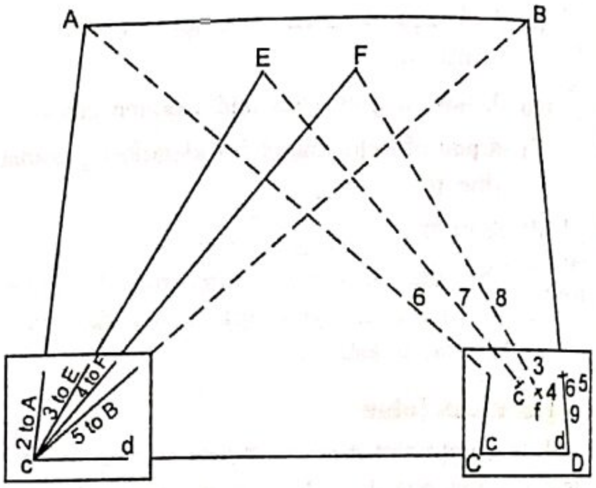

Fig. 320

The area to be surveyed is shown in Fig. 320. The points to be fixed on the drawing are *A*, *B*, *C*, and *D*, the four corners of the field and two other points *E* and *F*. All these stations can be marked by ranging rods. Hence *CD* can be selected as the base line and the point *C* as the starting station. Before fixing the table over the station, get it approximately level by eye judgment at some other point and then move it bodily over the station. Now place the spirit level parallel to one edge of the table and skillfully manipulate the legs to get the air-bubble of the level in mid-run. Next, place the spirit level parallel to another edge at right angles to the former and again adjust the level.

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When the area to be surveyed is large, and the scale is small the entire board will not be larger than a point in the drawing so that the starting station point can be taken anywhere on the board; but when the area to be surveyed is small and station point should be exactly located and marked on the table. For this is a long fork can be used and with the help of a plumb-bob the station point can be fixed exactly on the board. Ordinarily the station point could be marked by eye judgment. After marking the point *C* on the board to represent the station *C*, the side *CD* is chained carefully and length *cd* marked, according to suitable scale. The length and direction of the line *cd* should be chosen with discretion so that the whole area should come squarely on the board. Now, one edge of the alidade is placed along the line *c*—*d* and the table turned towards *D* until the hair line intersects the ranging rod at *D*. The table is clamped and now, the table is said to be *oriented*. One of the longer edges of the trough compass is now placed against the point *C* and the compass turned round until the needle assumes a normal position. A line is drawn along the edge touching *c* and the inclination of this line with *c*–*d* gives the magnetic bearing. The table is now said to be lying in *azimuth*. Rays now can be drawn to other points to be fixed on the board and these rays are labelled properly to avoid confusion. The board is removed to *D* and levelled up. The same edge of the alidade is placed along *cd* and after unclamping the board is rotated slowly until the ranging rod at *c* is intersected by the hair-line. The board is now clamped and it is again oriented. Rays are drawn to the points *A*, *B*, *E* and *F*, intersecting the corresponding rays, drawn from *C* at *a*, *b*, *c* and *f*, and thus, all the points are fixed to scale. If a point which is not visible from *C* and *D* is to be fixed, the table could be moved to some other station and the point fixed from there. Even otherwise, it is best to inersect every point, from at least three stations.

Suggestions

(1) The table should have a tight fit and should not shake during work.

(2) Orientation over a station should be done by back sights and the compass can be used only as a check.

(3) No pressure should be placed on the board while working and contact with the legs should be carefully avoided. The general tendency is to set the table too high. The board should be just below the elbow so that all parts could be reached easily.

(4) The alidade should have a perfectly straight edges and lines should be drawn close to the edge. If any part of the alidade is raised off the surface, the pencil should not run under it.

(5) The rays drawn should be as fine as possible. It is convenient to have a piece of sandpaper attached to the board by a thread. The rays need not be drawn full length. Carefully draw short line where the point is to be plotted. While intersecting, the second line need not be drawn at all—the point is only marked. For orientation long lines are necessary and for this, short lines can be drawn, at each end of the alidade.

(6) The rays drawn should be properly labelled so that there is no confusion while intersecting from a second station.

(7) As a matter of precaution every station should be intersected from three or four stations. Before leaving a station a check-sight should be taken to see if the table has been disturbed. Short distances could be measured in the field by chain or tape and the corresponding lengths should be verified on the drawing.

(8) The scale should be selected according to the amount of details to be shown but it is always convenient to choose some multiple of 10.

(9) When it is necessary to continue the survey on another sheet, a few stations from the first sheet should be transferred to the second by pricking or any other suitable methods.

Sources of Errors

(1) Errors due to the station point marked on the board being not exactly above the point on the ground.

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(2) Errors due to loose fitting and wrong levelling.
(3) Errors due to imperfect sighting.
(4) Errors due to lines not being drawn perfectly straight.
(5) Errors in measurement of the base line and in laying off the exact length.
(6) Errors due to changes of weather.
(7) Errors due to wrong labellng of the rays

The Three-Point Problem

Sometimes it is necessary to determine the position of a plane table in the field with the help of three objects the positions of which are already marked on the map. This interpolation is done by the method of resection commonly used for plane-table survey. There are three ways by which the problem can be solved :

(1) Mechanical method,
(2) Graphic method, and
(3) Trial method.

The basic problem is that the plane table is set-up at a point *P* in the field from where three object *A*, *B*, *C*, are visible. If the plotted points *a*, *b*, *c*, corresponding to the objects in the field are shown on the map, it is required to plot the point, *p*, corresponding to the position of the plane-table.

1. Mechanical Method

With the help of drawing-pins fasten a piece of tracing paper on the broad. Mark a point *p* on the tracing paper to represent the position of the table. Put one edge of the alidade against point *p* and take sights successively to the three objects *A*, *B*, *C* and draw long rays. Unpin the tracing paper and shift it until the rays pass through the correspoinding objects, *a*, *b*, and *c* on the map. Now repin the tracing paper and prick *p*, the point of intersection of the three rays, and the position of the table will be at the point where the pin-prick touches the map. Next place the edge of the alidade along *p* and orient the table by sighting at *A*. Draw rays joining *bB* and *cC* respectively and the three rays would meet at *p*. If there is a slighting inaccuracy, the lines form a small triangle which can be eliminated by slight movement of the board or the position can be found by the triangle of error method explained later.

2. Graphic Method

There are several graphic solutions to the three-point problem, but here only Llano's graphic method will be described.

The three objects *A*, *B* and *C* and their plotted positions are shown in Fig. 321. Draw a perpendicular line dividing *ab* into two parts. Place the alidade along this line and to rotate the table until *B* is sighted and then clamp the table. Place the edge of the alidade at the point *a* and to date the alidade until *A* is sighted and then draw then ray *A* to intersect the previous line at *d*.

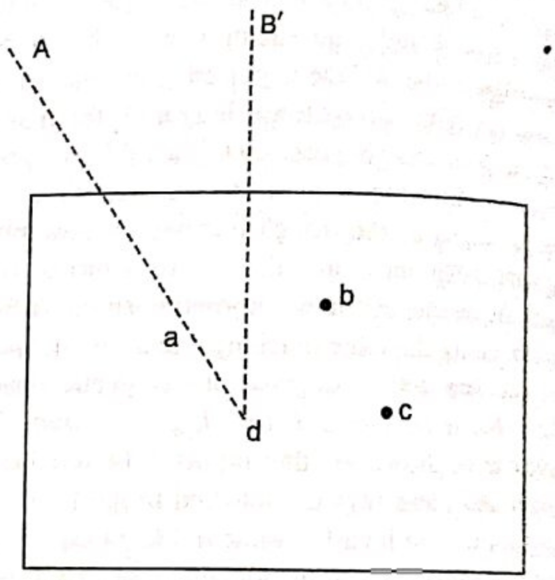

Fig. 321

Similarly bisect the line *bc* by a perpendicular and after placing the alidade along this line unclamped and rotate the table until *B* is sighted.

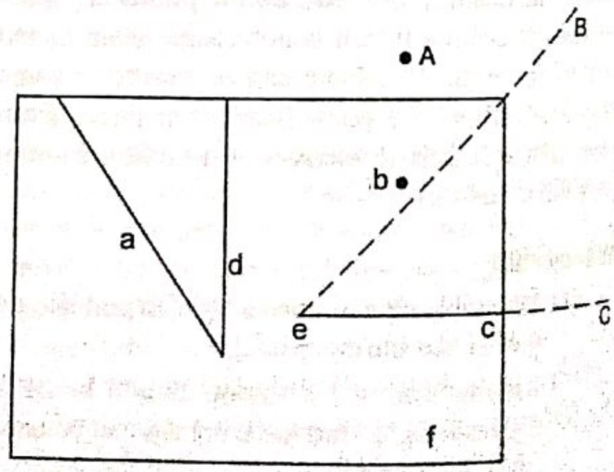

Fig. 322

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now clamp the table and draw the sight line cC to intersect at e. (See Fig. 322).

With centres d and c and radii ad and ec respectively draw arcs of circles to intersect at f, the required position of the plane-table on the map.

3. Trial Method

This method is also known as the triangle of error method.

Orient the table as far as possible by the help of the compass. Choose three visible objects in the field and find their corresponding positions on the map. These three objects should not be on the circumference of a circle. Now clamp the table and draw lines a, b and c by sighting at A, B and C respectively. If the table is by chance oriented properly, rays will meet at the required point P, but most commonly they will form a small triangle called the triangle of error. In case triangle of error is formed there are two alternatives. *Firstly*, if the position of the table is within the triangle formed, by imaginary lines joining the objects in the field, then the position p will be within by the triangle of error so that the perpendicular distance of such a position from any ray is proportional to the distance of the object from which that ray was drawn (See Fig. 323). *Secondly*, when the table is outside the imaginary triangle the position p of the table on the map will be outside the triangle of error either to the left or to the right of all the rays. Fig. 324 shows that p and p' are the only two such positons, where p is to the left of all the rays and p' to the right of all the rays. In any other sector this condition is not fulfilled. To determine whether the point should be on the left side or on the right side of the rays, unclamp the

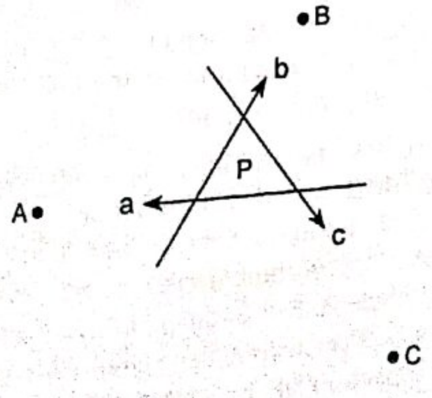

Fig. 323

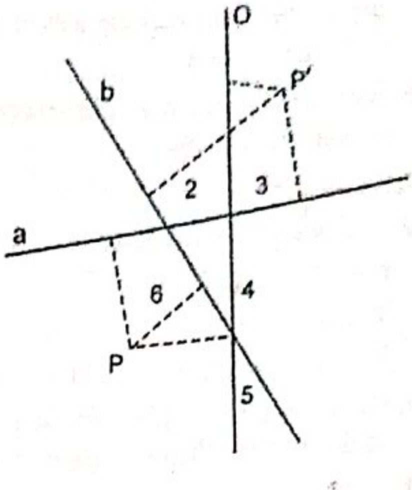

Fig. 324

table and rotate slightly to the left and after clamping draw new rays sightly at the objects and if the new triangle of error is larger than the first triangle of error, the position should be p', that is on the right of the rays, but if the new triangle of error is smaller than the first the position is p, that is on the left of the rays. After determining whether it is p or p' fix the position by drawing perpendiculars to the rays proportional to the distances of the rays from the objects. This rule that the perpendicular distance of such a position from any ray is proportional to the distance of the object from which that ray was drawn would also indicate whether the position would be p or p'.

Advantages of Plane-table Surveying

(1) There is no necessity of a field-book and thus mistakes in recording are completely avoided.

(2) The entire plotting is done in the field and thus there is less likelihood of over-looking any details which ought to be shown.

(3) In the method described it is necessary only to measure one base line and thus mistakes in measurement of lines or angles are the least.

(4) There is ample space for checking the survey with the progress of the work.

(5) By the method of resection the position of a point with respect to there known points can be easily determined. Thus the surveyor

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भारतीय क्लाइनोमीटर
(Indian Clinometer)

इस क्लाइनोमीटर का आविष्कार भारतीय सर्वेक्षण विभाग ने किया था, अतः इसे 'भारतीय क्लाइनोमीटर' नाम से पुकारते हैं। भारतीय क्लाइनोमीटर को टेन्जेन्ट क्लाइनोमीटर (tangent clinometer) भी कहा जाता है। अपेक्षाकृत दूर स्थित बिन्दुओं की ऊँचाइयाँ ज्ञात करने के लिये यह यन्त्र बहुत उपयोगी रहता है। भारतीय क्लाइनोमीटर को प्लेन टेबुल पर रखकर प्रयोग करते हैं तथा इसके द्वारा उन्नयन व अवनमन कोणों को मापा जाता है।

[I] भारतीय क्लाइनोमीटर के अंग
(Parts of an Indian clinometer)

जैसा कि चित्र 21.17 से स्पष्ट है, भारतीय क्लाइनोमीटर में निम्नलिखित प्रमुख अंग होते हैं :

1. **आधार प्लेट** (Base plate)—भारतीय क्लाइनोमीटर में सबसे नीचे लगभग 22 सेमी लम्बी एवं 2 सेमी चौड़ी धातु की प्लेट होती है, जिसे आधार प्लेट कहते हैं। इस प्लेट के नीचे हाथी दाँत अथवा धातु के तीन छोटे-छोटे बटन या पैर होते हैं। ये पैर आधार प्लेट को प्लेन टेबुल के कागज़ से थोड़ा ऊपर रखते हैं अतः उपकरण को प्लेन टेबुल पर घुमाने से कागज़ खराब होने से बच जाता है।

2. **पीतल की छड़** (Brass bar)—आधार प्लेट के ऊपर पीतल की एक छड़ होती है जिसके मध्य में **लेविल नलिका** एवं एक सिरे पर **समतलन पेंच** (levelling screw) लगा होता है। इस खंचित पेंच की सहायता से पीतल की छड़ को समतल किया जाता है। इस छड़ में दो **आलंबन** (strut) लगे होते हैं, जो कोण मापते समय क्लाइनोमीटर के नेत्र फलक (eye vane) व दृश्य वेधिका (object vane) को लम्बवत् रखते हैं। उपकरण को बन्द करते समय इन आलंबनों को उल्टा घुमाकर आधार प्लेट पर क्षैतिज रख देते हैं।

3. **नेत्र फलक व दृश्य वेधिका** (Eye vane and object vane)—पीतल की छड़ के सिरों पर एक दूसरे से लगभग 20 सेमी दूर दो मुड़वाँ फलक (folding vanes) होते हैं। इनमें छोटा फलक, जिसके ऊपरी सिरे के पास **अवलोकन-छिद्र** (eye-hole) होता है, **नेत्र-फलक** (eye vane) या **दर्श फलक** (sight vane) कहलाता है तथा बड़े फलक को **दृश्य वेधिका** (object vane) कहते हैं। दृश्य वेधिका के मध्य में एक लम्बवत् झिरी (slit) कटी होती है। इस झिरी के बायें किनारे पर अंशों (degrees) के तथा दायें किनारे पर प्राकृतिक टेंजेन्ट (natural

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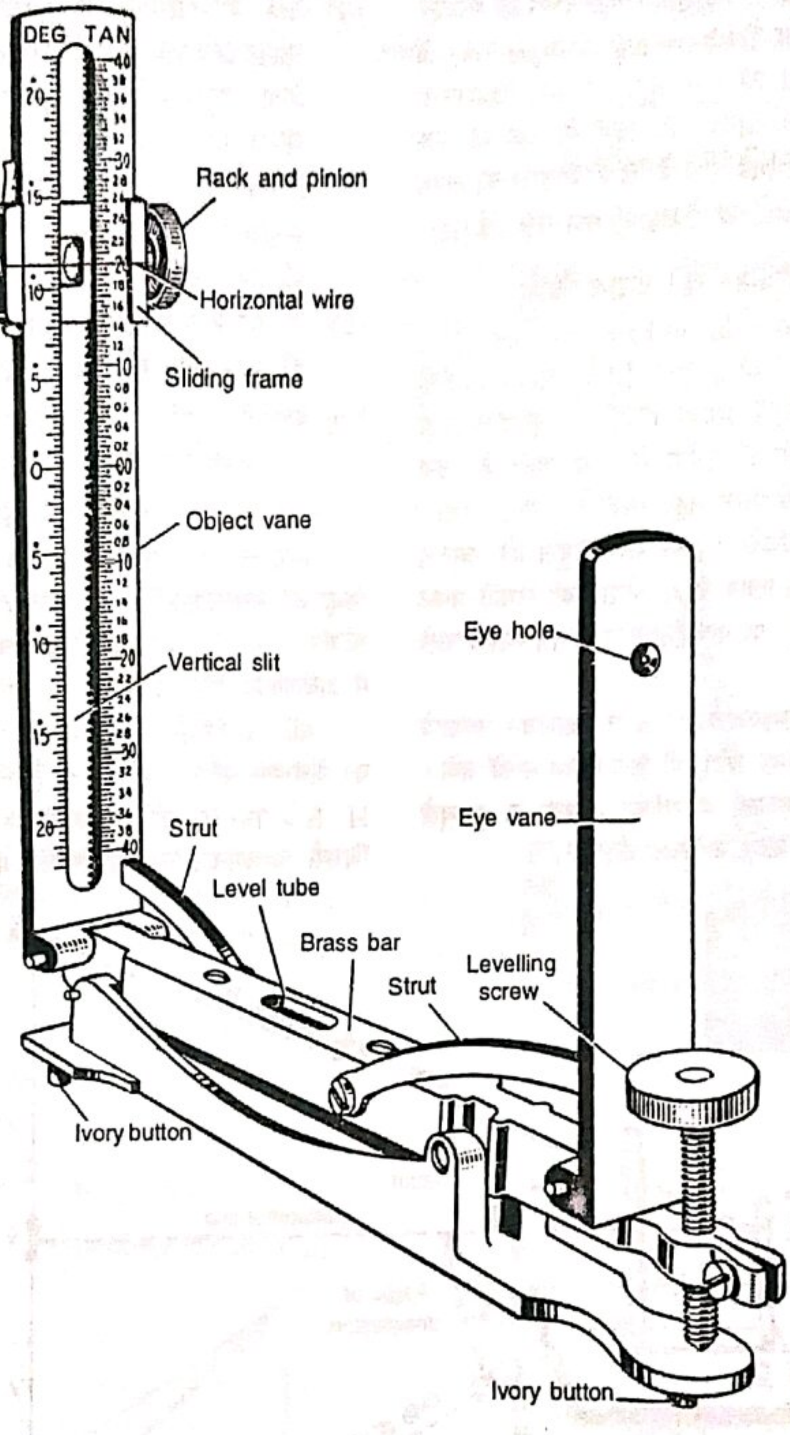

चित्र 21.17—भारतीय क्लाइनोमीटर।

tangents) के चिह्न अंकित होते हैं। दोनों किनारों पर मध्य में शून्य के चिह्न होते हैं। शून्य से ऊपर की ओर के चिह्नों पर उन्नयन कोण (angles of elevation) तथा नीचे की ओर के चिह्नों पर अवनमन कोण (angles of depression) पढ़ते हैं। इस प्रकार इस क्लाइनोमीटर पर अधिक से अधिक 22° तथा कम से कम 20 मिनट तक का उन्नयन या अवनमन कोण पढ़ा जा सकता है। इसी प्रकार दायें किनारे पर अधिक से अधिक 0.4 तथा कम से कम 0.005 टेंजेन्ट मूल्य पढ़ा जा सकता है। इस सम्बन्ध में यह बात उल्लेखनीय है कि दोनों मापनियों के शून्य चिह्न अवलोकन-छिद्र की सीध में होते हैं अर्थात् उपकरण को समतल स्थापित कर देने पर अवलोकन-छिद्र एवं शून्य के चिह्नों को मिलाने वाली कल्पित सरल रेखा पूर्णतः क्षैतिज हो जाती है।

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4. सरकवाँ फ्रेम (Sliding frame)—दृश्य वेधिका पर एक फ्रेम लगा होता है, जिसे **रैक-पिनियन** (rack and pinion) के द्वारा दृश्य वेधिका के सहारे ऊपर या नीचे की ओर खिसकाया जा सकता है। फ्रेम में दृश्य वेधिका की झिरी के आर-पार एक क्षैतिज तार बंधा होता है जिसकी सीध में किसी विवरण को लक्ष्य करके ऊर्ध्वाधर कोण का अंशों या टेंजेन्ट में मान पढ़ते हैं।

[II] भारतीय क्लाइनोमीटर की प्रयोग-विधि
(Method of using the Indian clinometer)

जैसा कि पहले संकेत किया जा चुका है, प्लेनटेबुलन में किसी बिन्दु की यन्त्र-स्टेशन से ऊँचाई अथवा नीचाई ज्ञात करने के लिये प्रायः भारतीय क्लाइनोमीटर को प्रयोग में लाया जाता है। इस उपकरण को प्रयोग करने की विधि बहुत सरल है (चित्र 21.18)।

(1) यन्त्र-स्टेशन (मान लीजिये A) पर प्लेन टेबुल को समतल स्थापित कीजिये तथा प्लान में A स्टेशन की स्थिति प्रकट करने वाले बिन्दु (a) पर क्लाइनोमीटर के नेत्र फलक वाले सिरे को रखिये।

(2) दोनों फलकों को लम्बवत् खड़ा करके आलंबन लगाइये जिससे कार्य करते समय कोई भी फलक नत न हो सके।

(3) समतलन पेंच की सहायता से लेविल नलिका के बुलबुले को नलिका के ठीक मध्य में स्थिर कीजिये।

(4) अब रैक-पिनियन से सरकवाँ फ्रेम को इतना ऊँचा खिसकाइये कि अवलोकन-छिद्र पर आँख रखकर देखने में प्रेक्षित किये जाने वाला विवरण (मान लीजिये B बिन्दु) क्षैतिज तार से प्रतिच्छेदित हो जाये। इसके पश्चात् दृश्य वेधिका पर क्षैतिज तार के सामने का चिह्न पढ़िये। यह पाठ्यांक ऊर्ध्वाधर कोण (मान लीजिये α) के टेंजेन्ट मूल्य को प्रकट करेगा।

(5) धरातल से अवलोकन-छिद्र की ऊँचाई तथा A व B के बीच की क्षैतिज दूरी को ज़रीब अथवा फीते से मापिये।

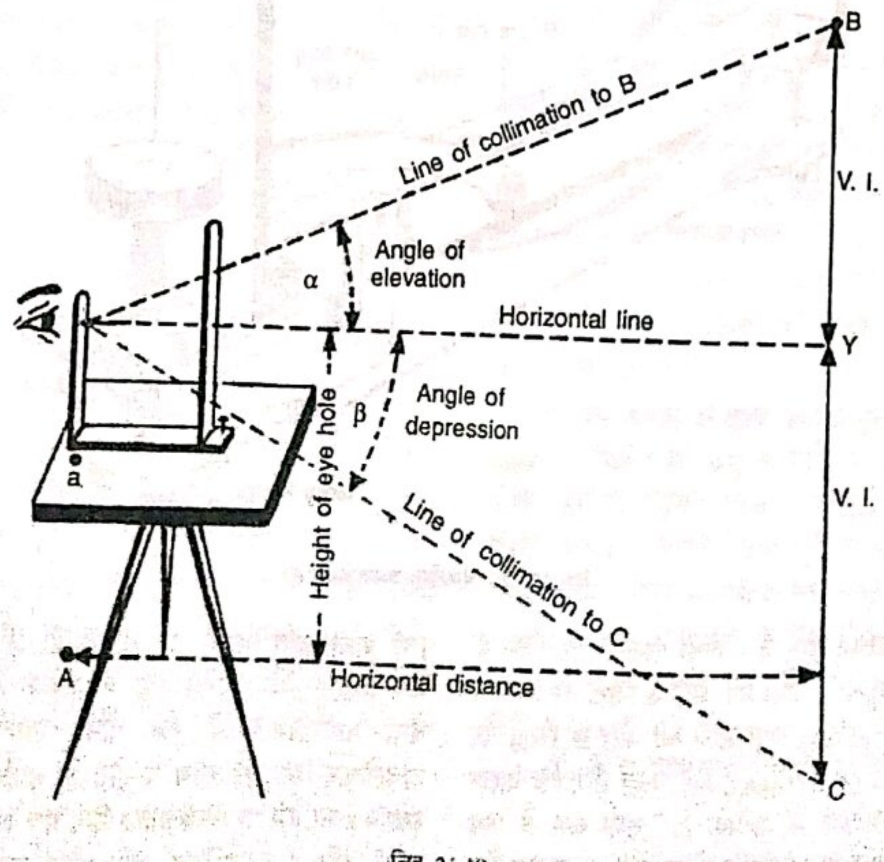
(6) उपरोक्त मानों को निम्न सूत्र में रखिये—

ऊर्ध्वाधर अन्तराल (V.I.),

$$= \text{क्षैतिज दूरी (H. E.)} \times \text{टैन}\ \alpha$$

जैसा कि चित्र से प्रकट है, इस सूत्र को हल करने पर B बिन्दु की अवलोकन-छिद्र से ऊँचाई (YB) ज्ञात होगी। अतः A स्टेशन से B बिन्दु की ऊँचाई ज्ञात करने के लिये YB के मान में अवलोकन-छिद्र की ऊँचाई को जोड़िये।

यदि A स्टेशन से नीचाई की ओर स्थित किसी बिन्दु C का अवनमन कोण β मापा गया है, तो उपरोक्त सूत्र (V. I. = H. E.× Tanβ) को हल करने पर YC का मान ज्ञात होगा, जिससे अवलोकन-छिद्र की ऊँचाई भी सम्मिलित है। अतः A



चित्र 21.18

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...से B बिन्दु की नीचाई ज्ञात करने के लिये YC के मान ...से अवलोकन-छिद्र की ऊँचाई को घटाया जायेगा।

(iii) दूरस्थ बिन्दुओं की ऊँचाई ज्ञात करना (Determining the heights of distant points)

जब कोई बिन्दु यन्त्र-स्टेशन से बहुत दूर स्थित होता है अथवा मार्ग में कोई बाधा होने के कारण क्षैतिज दूरी मापने में कठिनाई होती है, तो दो सरेख (collinear) स्टेशनों से पढ़े गये ऊर्ध्वाधर कोणों एवं उन स्टेशनों के बीच की क्षैतिज दूरी के आधार पर दिये हुए बिन्दु की यन्त्र-स्टेशन से ऊँचाई या नीचाई ज्ञात करते हैं। उदाहरणार्थ, मान लीजिये किसी यन्त्र-स्टेशन A से दिये हुए बिन्दु B की ऊँचाई ज्ञात करनी है परन्तु मार्ग में तालाब होने के कारण A व B के बीच की क्षैतिज दूरी को सरलतापूर्वक मापना सम्भव नहीं है (चित्र 21.19)।

(1) A स्टेशन पर प्लेन टेबुल को समतल स्थापित कीजिये तथा पहले बतलायी गई विधि के अनुसार क्लाइनोमीटर को प्लेन टेबुल पर रखकर B बिन्दु का ऊर्ध्वाधर कोण (α) ज्ञात कीजिये एवं अवलोकन-छिद्र की ऊँचाई मापिये।

(2) A स्टेशन से B की सीध में कुछ दूरी पर कोई दूसरा ऐसा स्टेशन C चुनिये जिसकी ऊँचाई A स्टेशन की ऊँचाई के समान हो। AC दूरी को मापिये।

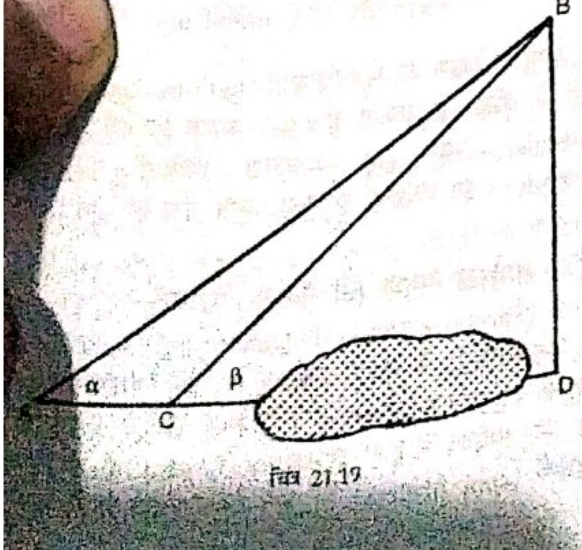
(3) C स्टेशन पर प्लेन टेबुल को समतल करके उस पर क्लाइनोमीटर को पहले के बराबर ऊँचाई पर रखकर B का पुनः ऊर्ध्वाधर कोण (β) ज्ञात कीजिये।

(4) उपरोक्त मानों को निम्नलिखित सूत्र में रखकर B बिन्दु की ऊँचाई ज्ञात कीजिये :

B बिन्दु की ऊँचाई,

$$= \frac{\text{AC दूरी} \times \text{टेन}\,\alpha \times \text{टेन}\,\beta}{\text{टेन}\,\beta - \text{टेन}\,\alpha}$$

± अवलोकन-छिद्र की ऊँचाई

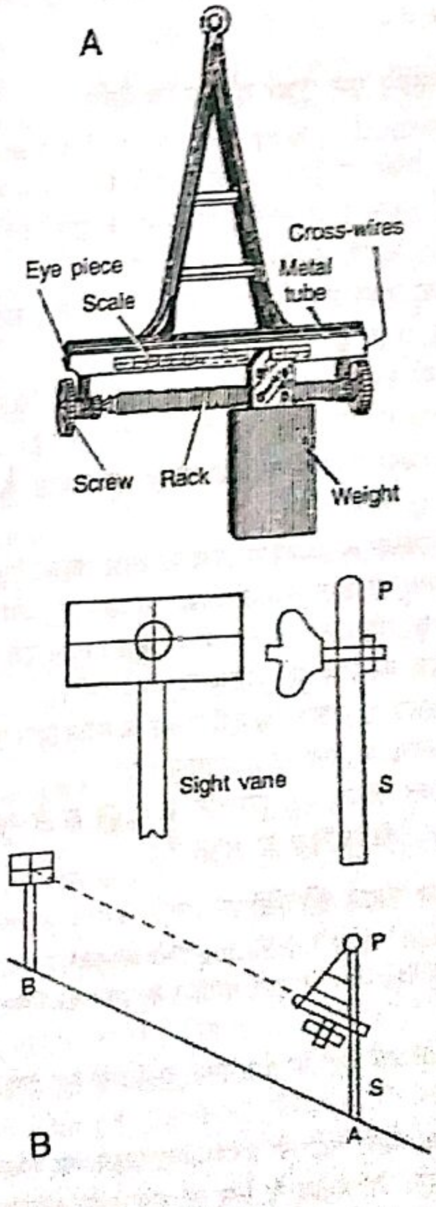


चित्र 21.19

स्मरण रहे, यदि प्रेक्षित बिन्दु यन्त्र-स्टेशन से नीचाई की ओर होता है तो अवलोकन-छिद्र की ऊँचाई को घटाया जाता है।

सीलॉन घाट ट्रेसर (Ceylon Ghat Tracer)

यह एक साधारण सर्वेक्षण यन्त्र है (चित्र 21.20 A)। पहाड़ी क्षेत्रों में किसी निश्चित ढाल वाली सड़क बनाने के लिये किये जाने वाले प्रारम्भिक सर्वेक्षण में तथा ढाल का कोण ज्ञात करने के लिये प्राय: इस उपकरण का प्रयोग लाभप्रद रहता है। सीलॉन घाट ट्रेसर में धातु की एक खोखली नलिका होती है। इस नलिका के एक सिरे पर अवलोकन-छिद्र (eye hole) होता है तथा दूसरे सिरे



(G-20)

चित्र 21.20

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